Why the Vacuum does not gravitate

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1 Fields as extra dimensions

Gradients are familiar in the actions of fields, though without an explanation of their origin. Special Relativity has answered a comparable question at the level of objects: in abstract spacetime $(t,\vec{x})$, where t is time and $\vec{x}$ is position in 3-space, live objects sweeping out concrete world lines $\vec{x}_n(t)$, the index numbering the object. The length of these world lines can be computed by - in this case rather rudimentary - Gauss' formula for the metric on embedded manifolds, what yields derivatives of $\vec{x}$ w.r.t. t if one simply choses t to parametrize the curve. The action of an object is the length of its world line multiplied by its mass.

The same mechanism escalated to the level of fields is: there is an abstract "embedding space" $(t,\vec{x},\Theta) \equiv (x^\mu,\Theta)$ with $\mu = 0,1,2,3$ and Θ denoting the entirety of fields, in which live concrete fields each sweeping out a 4-dimensional submanifold $\Theta_n(x^\mu)$. For example, for a sole vector field A (index supressed) there are 4 field equations $A^\mu(x^\nu)$, and all the non-vector fields set zero. The 4-volume of any such "field manifold" can be computed by Gauss' formula for the metric on embedded manifolds, once the metric of the embedding space has been specified. The action of the field is the 4-volume of its field manifold multiplied by some constant. So one is drawn to the conclusion that *the fields are extra dimensions attached to spacetime*.

Developing the idea for vector fields unveils an intimate connection to the Born-Infeld theory [1, 2, 3], as shall be discussed elsewhere. Here, the quintessence shall be demonstrated using scalars Φ despite of some problems associated. Mass and further self-interactions only can be added by hand in a clumsy way, just like potential terms are cumbersome in Special Relativity. But it is nothing new that the production of masses of field quanta requires additional concepts anyway. Regarding only scalars, a 6-dimensional embedding space is sufficient. A plausible metric reads, denoted in terms of the line element

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu \pm \frac{m_{pl}\ell_{pl}^5}{2}(\mathrm{d}\Phi\mathrm{d}\Phi^* + \mathrm{d}\Phi^*\mathrm{d}\Phi) \tag{1}$$

where m_{pl} is the Planck mass and ℓ_{pl} is the Planck length, respectively, with $\hbar \equiv m_{pl}\ell_{pl}$. The asterix means complex conjugation. The undetermined sign of the second term shall indicate that Φ can be spacelike (likely) or timelike. Furtheron, only one sign will be used, which eventually has to be brought in line with the sign convention for spacetime. Here and everywhere except exlicitly stated otherwise, the indices are moved by the metric of spacetime $g_{\mu\nu}$. Unfortunately, the conversion factor is not free of ambiguity. Without doubt, it is of Planckian order of magnitude, what means that Newton's constant $G \equiv \frac{\ell_{pl}}{m_{pl}}$ is involved, i.e. $m_{pl}\ell_{pl}^5 \equiv G^2\hbar^3$. But there could be some individual factor order of magnitude unity, as it turns out naturally for the vector fields.

The application of Gauss' formula yields for the induced metric γ on the 4-dimensional submanifold determined by the two field equations $\Phi(x^\mu)$ and $\Phi^*(x^\mu)$, where the index $n = 1$ of the one and only field regarded is supressed

$$\gamma_{\mu\nu} = g_{\mu\nu} + \frac{m_{pl}\ell_{pl}^5}{2}(\partial_\mu\Phi\partial_\nu\Phi^* + \partial_\mu\Phi^*\partial_\nu\Phi) \tag{2}$$

The determinant of γ is

$$\det\gamma = (\det g)\left(1 + m_{pl}\ell_{pl}^5\partial^\mu\Phi\partial_\mu\Phi^*\right) + O(m_{pl}^2\ell_{pl}^{10})\,. \tag{3}$$

So the action of a single concrete scalar field, which is the 4-volume of the field manifold up to the appropriate forefactor reads up to the fifth power in the Planck length

$$\mathscr{S} = \frac{m_{pl}}{\ell_{pl}^3}\int\sqrt{-\det\gamma}\,\mathrm{d}^4x \approx \int\left(\frac{m_{pl}}{\ell_{pl}^3} + \frac{1}{2}\hbar^2\partial^\mu\Phi\partial_\mu\Phi^*\right)\sqrt{-\det g}\,\mathrm{d}^4x\,. \tag{4}$$

2 Field equations

The general formula for the derivative of any nonvanishing determinant reads

$$\frac{\partial\det\gamma}{\partial x} = (\det\gamma)\,\gamma^{\hat{k}\hat{l}}\frac{\partial\gamma_{kl}}{\partial x}\,, \tag{5}$$

where the hat on the indices shall indicate that these clearly are moved by γ, i.e. $\gamma_{\hat{k}\hat{l}}\gamma^{\hat{l}\hat{m}} = \mathbb{1}_4$ while $\gamma_{\hat{k}\hat{l}} \equiv \gamma_{kl}$ remains unchanged. Furthermore, we have

$$\gamma^{\hat{k}\hat{l}}\frac{\partial\gamma_{kl}}{\partial\partial_\mu\Phi^*} = \gamma^{\hat{k}\hat{l}}(\delta_k^\mu\partial_l\Phi + \delta_l^\mu\partial_k\Phi) = 2\partial^{\hat{\mu}}\Phi\,, \tag{6}$$

where there now is a hat on μ.

This yields the field equation in the familiar form

$$\partial_\mu \left(\sqrt{-\det \gamma}\, \partial^{\hat{\mu}} \Phi \right) = 0\,. \tag{7}$$

The second field equation is just the complex conjugate. Both can be combined as usual to

$$\begin{aligned} \Phi^* \partial_\mu \left(\sqrt{-\det \gamma}\, \partial^{\hat{\mu}} \Phi \right) - \Phi \partial_\mu \left(\sqrt{-\det \gamma}\, \partial^{\hat{\mu}} \Phi^* \right) = \\ \partial_\mu \left[\sqrt{-\det \gamma} \left(\Phi^*\, \partial^{\hat{\mu}} \Phi - \Phi \partial^{\hat{\mu}} \Phi^* \right) \right] = 0\,. \end{aligned} \tag{8}$$

3 Broken symmetry

The field equations reflect the symmetry of the line element (1). All the references are to the internal metric γ, there is no reference to the metric g of spacetime, where the observers live. But this is not the whole story. In Special Relativity, the line element reads $\mathrm{d}\tau^2 = \mathrm{d}t^2 - \mathrm{d}\vec{x}^2$, and the position $\vec{x}$ of any object has no influence on time t. Here, in contrast, the fields Θ_n (n running from 1 to the total number of fields) determine something that happens inside spacetime x^μ. This means that spacetime is a preferred frame of reference inside the 6-dimensional embedding space. Only if two different universes, each having its own position and orientation inside the embedding space, collided, the symmetry would become restored.

Since the frame is preferred, so is the stress-energy associated with it. The cosmological term is locked to our universe and its value cannot be changed by any coordinate transformation. So it can be subtracted from the action of the fields, whereas the nonlinearities remain. The action for a sole scalar reads, much like Born and Infeld had proposed for vectors on a phenomenological basis

$$\mathscr{S}_{observed} = \frac{m_{pl}}{\ell_{pl}^3} \int \left(\sqrt{-\det \gamma} - \sqrt{-\det g} \right)\, \mathrm{d}^4 x \approx \int \frac{1}{2} \hbar^2 \partial^\mu \Phi \partial_\mu \Phi^* \sqrt{-\det g}\, \mathrm{d}^4 x\,. \tag{9}$$

This gives a clear explanation why the vacuum does not gravitate when the Einstein-Hilbert Lagrangian is added.

4 Relation to second quantization

The cosmological term is nothing but the vacuum energy density of the fields, above quasi approached through the backdoor on a “classical” route. Currently the tension is low, since noone knows the actual values. But there is only consistency of the big picture if the value appearing in equation (4) is *exactly the same* as is derived from quantum physics of the respective field. Is this realistic?

It was already discussed that the mechanism here is an escalation of the mechanism underlying Special Relativity, from the physics of objects living in time to the level of fields living in spacetime. From this similarity originates one very strong indication that the above sought identity actually is fulfilled. The role of the cosmological term here is comparable to the role of the rest mass term in Special Relativity. When a harmonic oscillator is treated relativistically in a fully consistent manner, this leads to a SO(2,1) symmetry, what directly progresses to the level of quantization: not the Heisenberg algebra is the quantum algebra of a relativistic oscillator, rather SO(2,1) is. So, upon quantization, it turns out that mass simply is the ground state energy of the oscillator and is a quantum number in units of the energy quantum proportional to the frequency parameter [4, 5, 6] .

5 Conclusion

The idea of extra dimensions has been around for more than a century, without final conclusion. But the underlying assumption has always been that these extra dimensions are like spacetime in the essence, only with high extrinsic or intrinsic curvature. But Special Relativity taught that things could be more subtle: the position of an object is something very much different from time. Here, spacetime and fields living on it are unified like space and time are unified there. The mechanism explaining where the derivative terms in the actions come from is almost compelling, hence it could be wise to follow up on it. Scalars and vectors can be treated well, whereas gravitation and fermions require additional considerations.

References

[1] Born M, Infeld L, Proc. R. Soc. Lond. **A 144**, 255 (1934)

[2] Yang Y, *Solitons in Field Theory and Nonlinear Analysis*, Springer Monographs in Mathematics, New York (2001)

[3] Ellis J et al, Phys. Rev. **D 96**, 086012 (2017)

[4] Hecht KT, **Quantum Mechanics**, Springer-Verlag, New York (2000)

[5] Aldaya V, Bisquert J, Navarro-Salas J, Phys. Lett. **A 156**, 381 (1991)

[6] Simon DS, Phys. Lett. **A 375**, 3751 (2011)