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Pending Symmetry involving Newton's Constant

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Abstract This paper argues that there is a symmetry mediated by Newton's constant based on a Hermitean line element in an 8-dimensional space. The resulting relations for the electromagnetic field are those of the initial Born-Infeld theory, with the scale factor becoming purely imaginary and being determined by the Planck mass. Various specific aspects are discussed, including the role of vacuum energy density and structural similarity to Special Relativity.

Keywords Fundamental symmetries · Newton's constant · Initial Born-Infeld Theory

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1 Introduction

The most fundamental symmetries of nature are those mediated by the elements of the Planckian set of units: the velocity of light c, Planck's constant $\hbar$ and Newton's constant G. Special Relativity shows that spatial distances are to be converted to temporal distances in a uniquely defined way by means of the line element $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2$, where t is time and x is position in 3-space. Statistical physics in union with quantization teaches that physical quantities are to be expressed as pure numbers in a uniquely defined way by means of the phase space volume form $\mathrm{d}\Theta = \frac{\mathrm{d}x\wedge\mathrm{d}p}{\hbar}$, where p is momentum.

The relations involving G originating from General Relativity look much less transparent. Furthermore, gravitation and quantum physics cannot be unified on the basis of the current theories. This leads to the conjecture that there exists a yet undiscovered symmetry mediated by G.

In the following section, such a new symmetry is proposed and will turn out to imply the initial Born-Infeld theory formulated in 1934, apart from the value of the scale parameter. Then the two theories are compared and some related aspects are discussed.

2 A new symmetry involving G

In the following, units are employed where the velocity of light is unity, except where explicitly noted otherwise. Hence G and $h \equiv 2\pi\hbar$, have dimensions $\frac{length}{energy}$ or equivalent, and $length \cdot energy$ or equivalent, respectively.

Space-time can be characterized by a position vector. So, energy-momentum should enter the symmetry relation as a vector as well. One can find the appropriate candidate from the minimal electromagnetic coupling $p - eA$, where e is the elementary charge (of either sign), and A is the electromagnetic vector field. It can be clarified that this paper deals with the space-time aspects alone, thus the phenomena of nonabelian fields emerging from their internal symmetries are not covered and the discussion can be restricted to the electromagnetic field. Whether or not the elementary charge is that of an electron or $\frac{1}{3}$ of this value shall be left open.

The proposed symmetry is based on the Hermitian line element

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu \pm \imath\, G\, \mathrm{d}x_\mu \wedge \mathrm{d}\, eA^\mu \ , \tag{1}$$

where indices are raised and lowered due to the space-time metric, g, which can correspond to intrinsic curvature. The relative sign is not important, since there are two possible charge signs and the parameters will only appear squared. This line element implies an 8-dimensional space spanned by the space-time degrees of freedom plus the a priori independent vector field degrees of freedom carrying dimension of *energy* after multiplication by e. The metric in this 8-dimensional space is the tensor product

$$\Gamma = g \otimes \begin{pmatrix} 1 & \mp\imath G \\ \pm\imath G & 0 \end{pmatrix} \ . \tag{2}$$

The second factor of Γ can be expressed as a sum over Pauli-matrices σ_i and the 2×2 unit matrix $\mathbb{I}_2 \equiv \sigma_0$. In natural units ($G = 1$)

$$\begin{pmatrix} 1 & \mp\imath \\ \pm\imath & 0 \end{pmatrix} = \frac{1}{2}\sigma_0 \pm \sigma_2 + \frac{1}{2}\sigma_3 \; . \tag{3}$$

This can be interpreted as a spin $(\frac{1}{2}, \frac{1}{2})$ representation of the Lorentz group (which has nothing to do with the Lorentz group of space-time). The corresponding vector is $\mathrm{col}(\frac{1}{2}, 0, \pm 1, \frac{1}{2})$, which is "space"like. The line element (1) is invariant under Lorentz transformations changing this vector, while $\begin{pmatrix} \mathrm{d}x \\ \mathrm{d}eA \end{pmatrix}$ transforms correspondingly under the $(\frac{1}{2}, 0)$ representation.

3 Field equations emerging from the new symmetry

The four field equations $A^\nu(x^\mu)$ define a 4-dimensional sub-manifold $\mathscr{M}$ of the 8-dimensional space, where the metric γ induced on $\mathscr{M}$ can be computed from Gauss' expression $\gamma_{\mu\nu} = \Gamma_{ab} \frac{\partial X^a}{\partial x^\mu} \frac{\partial X^b}{\partial x^\nu}$. Therefore,

$$\gamma_{\mu\nu} = g_{\mu\nu} \pm \imath e G F_{\mu\nu} \; . \tag{4}$$

Extremizing the $\mathscr{M}$ 4-volume $\int \sqrt{-\det \gamma}\, \mathrm{d}^4 x$ yields the electromagnetic field action, including its coupling to g. From this and from the above equation it can be seen that the resulting theory is nothing but the Born-Infeld "New Field Theory" [1,2] with their parameter b (reference scale for the square root of the energy density)

$$b = \pm \frac{\imath}{eG} \; . \tag{5}$$

Subtracting the vacuum energy density appearing quasi classically without reference to second quantization, the action of the electromagnetic field reads

$$S = \frac{1}{e^2 G^2} \int \left[\sqrt{1 - \frac{e^2 G^2 F^2}{2} - \frac{(e^2 G^2 F F^*)^2}{16}} - 1 \right] \sqrt{-\det g}\, \mathrm{d}^4 x \, , \tag{6}$$

where F is the anti-symmetrized gradient of A, and F^* is its Hodge dual. Only ordinary space-time derivatives are involved due to F anti-symmetry and Christoffel symbols' symmetry. Products of the fields involve g as usual.

4 Comparison to Born-Infeld Theory

Regarding the Born-Infeld (BI) action in the current notation [2], different b values will alter the order of magnitude of the effects, whereas the imaginary unit toggles the sign for the F^2 term, consequently toggling the overall sign.

Regarding signs, the original BI Lagrangian, including vacuum energy density, is approximately proportional to $-b^2 \sqrt{1 + \frac{F^2}{2b^2}} = -b^2 \sqrt{1 + \frac{B^2 - E^2}{b^2}}$, where b is real.

B and E are the magnetic and the electric field, respectively. Therefore, if the field is primarily electrical, i.e., $E^2 > B^2$, then the overall minus sign is consistent, since total vanishing of the field corresponds to a maximum of the absolute value of the Lagrangian. Since b now is purely imaginary, the proposed theory has the signs flip to a Lagrangian approximately proportional to $|b|^2 \sqrt{1 - \frac{F^2}{2|b|^2}} = |b|^2 \sqrt{1 + \frac{E^2 - B^2}{|b|^2}}$, with a new value for $|b|$. If the field is primarily electrical, then the overall plus sign is consistent, since the total vanishing of the field corresponds to a minimum of the Lagrangian.

Born and Infeld's main aim was to remove difficulties arising at length scales of the electron radius, where the action emerged from considering invariance under the choice of space-time coordinates. Apart from a numerical factor of order of magnitude unity, BI estimated the scale from the electron mass , m_{e_-}, and electron radius, r_{e_-} as $|b| = \sqrt[+]{\frac{m_{e_-}}{r_{e_-}^3}} \propto \frac{m_{e_-}^2}{|e|^3}$ by equating the total energy of the field generated by the electron to its mass. However, although using the electron mass as the scale had become almost straightforward by the 1930s, nowadays a number of other elementary particles are known to have the same charge but different mass, leptons in particular. The Planck mass as the scale originating from the new symmetry expressed through the line element (1) is the plausible way out. But, despite of the equivalence apart from one constant, the philosophies behind the BI-theory and behind the theory proposed here are radically different.

It can be added that BI theory has been employed in a number of contexts, e.g. [3–8] and references therein, while the initial BI theory has remained of interest due to its structural plausibility [9].

5 Potential of a point charge

In paragraph 6 of [1], the potential Φ, whose spatial gradient is the electric field, was estimated for a point charge at rest in a flat background. Let us re-express the equations (6.8) of [1], replacing e by Ne (and inserting an absolute value symbol where appropriate), $N \in \mathbb{Z}$ to emphasize that the expressions hold for arbitrary charge that is an integer multiple of e,

$$\Phi(r) = \frac{Ne}{r_0} f\left(\frac{r}{r_0}\right) \; ; \; f(x) = \int_x^\infty \frac{\mathrm{d}y}{\sqrt[+]{1+y^4}} \; ; \; r_0 = \sqrt{\left|\frac{Ne}{b}\right|} , \qquad \text{(from [1], 6.8)}$$

where r is the distance from the source. Inserting the new scale yields

$$r_0 = |e|\sqrt{|N|G} = \sqrt{|N|\alpha}\; \ell_{pl} \; , \tag{7}$$

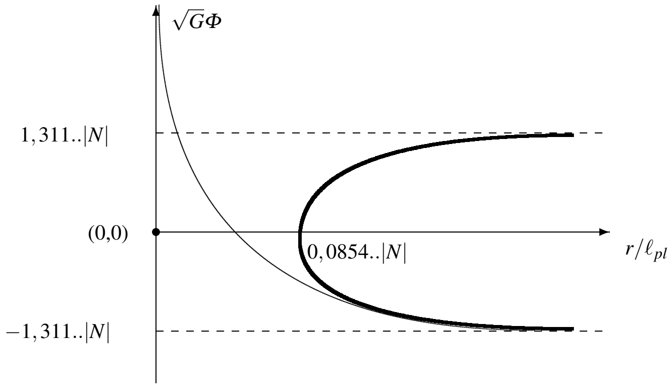
where α is the fine structure constant and $\ell_{pl} \equiv \sqrt{G\hbar}$ is the Planck length. Furthermore, due to the flip of the sign of the F^2-term, the integral is now

$$f(x) = \int_x^\infty \frac{\mathrm{d}y}{\sqrt[+]{y^4 - 1}} \; , \tag{8}$$

i.e., the potential has infinite slope but finite value at $r = r_0$. There is no potential in the interior, i.e., $r < r_0$, but rather there is a second branch in the outer region corresponding to the charge of opposite sign. Asymptotically, $f(x) \to \frac{1}{x}$, but always lies above this hyperbola, and can be transformed into Bernoulli's elliptic integral. The numeric value at the boundary is $f(1) = \frac{\varpi}{2} = 1,311...$, where ϖ is the (first) lemniscate constant. It makes sense to define the potential to be zero at r_0 and let its asymptotic values be $\pm\frac{\varpi}{2}$.

Figure 1 shows the two branches of Φ with the normalization $\Phi(r_0) = 0$, together with one asymptotic hyperbola. The horizontal scale is stretched by a factor of approximately 20 relative to the vertical scale, and only is approximately linear.

Fig. 1 The two branches of the potential of a point source of charge $\pm|Ne|$



6 Vacuum energy density

As emphasized in Section 3, the vacuum energy density ρ_{vac} appears without reference to second quantization. It has a comparable role here to rest energy $\mathscr{E}_{rest}$ in particle physics. It has Planckian order of magnitude, with the inverse fine structure constant being equivalent to rest mass m. We can show the correspondences with Special Relativity by setting $eF \equiv \mathscr{F}$, modifying the respective expression in the Lagrangian to $\frac{1}{e^2G^2}\sqrt{1 - \frac{G^2\mathscr{F}^2}{2}}$ with correspondences as shown in Table 1 in terms of absolute values, and signs were discussed above (Section 4). ρ_{pl} is the Planckian energy density, that is one Planck mass per Planck length cubed. The vacuum energy density of the electromagnetic field is about 137 times larger.

Since such energy density does not gravitate, it is subtracted in (6) following [1], which does not change the electromagnetic dynamics. However, the subtraction im-

Table 1 Constant correspondences

Particle physics	Electromagnetic field physics ($c=1$)
c	$\frac{\sqrt{2}}{G}$
m	$\frac{1}{2e^2} \equiv \frac{1}{2\alpha\hbar}$
$\mathscr{E}_{rest} = mc^2$	$\rho_{vac} = \frac{1}{e^2G^2} \equiv \frac{1}{\alpha}\rho_{pl}$

plies there are two unique manifolds: $\mathscr{M}$ and the vacuum, $\mathscr{V}\ :\ A^\mu = 0\ \forall\ \mu$. If the Riemannian (real, symmetric) component of the metric (2) is regarded separately, then it prefers the 4 space-time like directions in the embedding space, since increments in the A directions do not count. This degeneracy originates from pure helicity 1 of the electromagnetic field, and is intimately related to the vacuum definition. The constants defining the vacuum are zero because the field must be square-integrable. Hence the observed cosmological constant does not originate from the Planckian sized vacuum energy densities due to mutual cancellation of $\mathscr{M}$ and $\mathscr{V}$ contributions.

The existence of the vacuum originating from the degeneracy of the real part of the line element (1) implies our world is asymmetric, i.e., we exist in space-time not energy-momentum. However, the full embedding space metric does not have such degeneracy, as discussed in section 1, so there is a symmetry breaking involved.

Consider that $\mathscr{V}$ and $\mathscr{M}$ can be addressed as branes, and previous studies (e.g. [3,4]) have established a connection between BI theory, strings, and branes. The proposed theory introduces a new physical interpretation for the space branes occupy: 8-dimensional space-time-energy-momentum, where embedded 4-branes associated with fields other than the gravitational field can be regarded as space-time like. This could be important to interpret relationships, such as how massive particle worldlines are time like structures embedded in space-time.

7 Nonlinearity magnitudes

In principle, the proposed theory allows a test by observing electrodynamic nonlinearities. However, this has the standard problem of modern physics, i.e., the Planck mass is huge compared to currently achievable energy scales, being more than 22 orders of magnitude larger than the electron mass, and appearing squared in $|b|$.

The inverse magnitude of the nonlinearity is $\frac{c^4}{|e|G} = 2{,}5 \cdot 10^{58}\sqrt{\frac{g}{cm \cdot sec^2}} = 7{,}5 \cdot 10^{62}\,\frac{Volt}{meter}$ (or a factor 3 larger in eventum). Although current observations [9,10] can be precise enough to falsify the original BI scale, which is no longer surprising, about 40 orders of magnitude remain to reach the Planck scale (in mass squared).

What regards length scales, $r_0 < \ell_{pl}$ for $|N| < \frac{1}{\alpha}$, i.e., for all known elementary particles. Lengths below the Planck length are very questionable, however the much more extreme value is that of the Schwarzschild radius of, say, the electron. The very appearance of Newton's constant in the second factor of equation (2) makes obvious an intimate connection to gravitation, and implies that final answers only can be hoped for if gravity is taken into account.

8 Gravitation

Similar to BI, the gravitational field Lagrangian itself is not considered. However, it is necessary to discuss whether the proposed new symmetry can be compatible with gravity.

In equation (2), G appears without any reference to gravitation, which is likely to be the reason why this symmetry has not yet been conjectured. Rather, the embedding space metric contains the space-time metric as a separate factor, what can make sense. On the one hand, G appears as more than the constant of gravitation, like c is more than just the velocity of light. On the other hand, the space-time metric as a separate factor pre-curving the embedding space, is fully in the spirit of General Relativity. The resulting coupling of the gravitational field to the electromagnetic field appears as appealing, even though Pauli was unhappy with it [11].

As a conclusion, there is no conflict of this theory with gravitation. With the words of [1], it is easy to include the field action of the gravitational field by adding the respective Lagrangian. This can be the Einstein-Hilbert Lagrangian or a more sophisticated expression like Born-Infeld gravity [12]. This is good pragmatism, however peaceful coexistence of two symmetries mediated by G cannot be the ultimate wisdom. Actually, there can only be one. So the two factors in (2) eventually have to be reckognized as two sides of the same coin. This shall be discussed elsewhere.

That gravitation cannot be left out of consideration in either way, is clear from some nonsensical results of the Born-Infeld theory for any value of the parameter b. r_0 from equation (7) can be regarded as the radius of the charged source. Since at r_0 no other parameters than electric charge appear, the electron, μ, and τ all have the same radii. The same holds for the energy of the associated electric field, which is $\mathscr{E} \propto \sqrt{\frac{\alpha}{|N|}} m_{pl}$, where $m_{pl} \equiv \sqrt{\frac{\hbar}{G}}$ is the Planck mass, apart from a constant factor of order of magnitude unity (see [1], equation (8.6)).

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