# Backdoor to the Vacuum

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**Abstract**

At the example of scalars it is argued that (bosonic) fields are extra dimensions attached to spacetime. As a basis, Newton's constant appears inside a metric structure. The action contains a huge cosmological (vacuum) term, which however does not gravitate due to a broken symmetry. The relations to the principles of second quantization are discussed.

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Nur eine Waffe taugt:
Die Wunde schließt der Speer nur, der sie schlug.
(Wagner: Parsifal)

# 1 Fields as extra dimensions

The gradients appearing in the actions of fields are a consequence of the quantum postulate saying that momentum in the theory of objects becomes a spacetime derivative in the theory of fields. This has worked well, but remains somewhat enigmatic. Isn't there a fully consistent geometric picture where these derivatives appear naturally? Special Relativity - which to some degree can be adressed as zeroth quantization - has answered a comparable question at the level of objects: in abstract spacetime $(t,\vec{x})$, where $t$ is time and $\vec{x}$ is position in 3-space, live objects sweeping out concrete world lines $\vec{x}_n(t)$, the index numbering the object. The length of these world lines can be computed by - in this case rather rudimentary - Gauss' formula for the metric on embedded manifolds, what yields derivatives of $\vec{x}_n$ w.r.t. $t$ if one simply choses $t$ to parametrize the curve. The action of an object not subjected to external forces is minus the length of its world line multiplied by its mass. The metric of spacetime contains the velocity of light $c$ as a conversion factor, which will be set unity in the following.

The same mechanism escalated to the level of bosonic fields is: there is an abstract "embedding space" $(t,\vec{x},\Theta) \equiv (x^\mu,\Theta)$ with $\mu = 0,1,2,3$ and $\Theta$ denoting the entirety of abstract fields, in which live concrete fields each sweeping out a 4-dimensional submanifold. For example, for a sole vector field $A$ (index supressed) there are 4 field equations $A^\mu(x^0,x^1,x^2,x^3)$, and all the non-vector fields set zero. The 4-volume of any such "field manifold" can be computed by Gauss' formula for the metric on embedded manifolds, once the metric of the embedding space has been specified. The action of the field is the 4-volume of its field manifold multiplied by some constant. So one is drawn to the conclusion that *the fields are extra dimensions attached to spacetime*.

Developing the idea for vector fields unveils an intimate connection to the Born-Infeld theory [1, 2, 3], as shall be discussed elsewhere. Here, the quintessence shall be demonstrated using scalars $\Phi$ despite of some open questions associated. At first, the fields shall be massless, just like the introduction of potential terms is cumbersome in Special Relativity. Regarding only scalars, a 6-dimensional embedding space is sufficient. A plausible metric reads, denoted in terms of the line element

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu - \frac{G^2\hbar^3}{2}(\mathrm{d}\Phi\mathrm{d}\Phi^* + \mathrm{d}\Phi^*\mathrm{d}\Phi) \; , \tag{1}$$

where the appearance of the third power of Plancks (reduced) constant $\hbar$ is familiar from field theory. It originates from the fact that spacetime is 4-dimensional rather than 1-dimensional. The way Newton's constant $G$ appears is innovative: it does not simply act as a conversion factor like in the Einstein equations, rather it does so inside a (generalized, for the case of vectors) metric structure like $c$ does inside the metric of spacetime. Some purely numerical factor order of magnitude cannot be excluded, rather shall be included in $G$ by definition. The asterix means complex conjugation. The sign convention for spacetime is $(+---)$, so $\Phi$ is spacelike. This will be justified later. Here and everywhere except exlicitly stated otherwise, the indices are moved by the metric of spacetime $g_{\mu\nu}$.

In this 6-dimensional abstract space shall be embedded the field manifold of one concrete massless scalar field indexed $n$ by means of the two field equations $\Phi_n(x^0,x^1,x^2,x^3)$ and $\Phi_n^*(x^0,x^1,x^2,x^3)$. However, one aspect needs clarification first. The quintessence of the line element (1) is a relation $\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu - G^2\mathrm{d}p^2$, where $p$ is momentum (or energy). Though multiplication by $\sqrt{\hbar^3}$ assigns to a scalar field the dimension of energy, this is just dimensional analysis. It is not ultimately sure that the result is identic with an entity that actually has to be measured as such in the concrete case. In other words, for any concrete scalar there could be an individual scaling constant order of magnitude unity, which shall be denoted as $\beta$.

With this, Gauss' formula for the induced metric $\gamma_n$ on the 4-dimensional submanifold yields

$$\gamma_{n\mu\nu} = g_{\mu\nu} - \frac{\beta_n^2 G^2\hbar^3}{2}(\partial_\mu\Phi_n\partial_\nu\Phi_n^* + \partial_\nu\Phi_n\partial_\mu\Phi_n^*) \; . \tag{2}$$

The determinant of $\gamma_n$ is

$$\det\gamma_n = (\det g)\left(1-\beta_n^2 G^2\hbar^3\partial^\mu\Phi\partial_\mu\Phi^*\right)+O(G^4\hbar^6)\ . \tag{3}$$

So the action of a single concrete scalar field, which is proportional to the 4-volume of the field manifold reads

$$\mathscr{S} = -\frac{1}{\beta_n^2 G^2\hbar}\int\sqrt{-\det\gamma_n}\,\mathrm{d}^4x \approx \int\left(-\frac{1}{\beta_n^2 G^2\hbar}+\frac{1}{2}\hbar^2\partial^\mu\Phi_n\partial_\mu\Phi_n^*\right)\sqrt{-\det g}\,\mathrm{d}^4x\ . \tag{4}$$

The cosmological term has a negative sign, hence the vacuum energy density is positive as it should be for bosons.

To some extent, it is close to trivial, that realtions like equation (1) lead to actions like equation (4) if the metric and the parameters inside it are chosen appropriately.

# 2 Field equations

Definition and a caveat: in this section, the indices are moved by virtue of $\gamma$ rather than $g$.

The approximate field equations derived from equation (4) obviously are as usual. However, they also can be formulated exactly, essentially by letting $\gamma$ act in place of $g$. In the following the index of the concrete field $\Phi$ is supressed. The general formula for the derivative of any nonvanishing determinant reads

$$\frac{\partial\det\gamma}{\partial x} = (\det\gamma)\,\gamma^{kl}\frac{\partial\gamma_{kl}}{\partial x}\ . \tag{5}$$

Furthermore

$$\gamma^{kl}\frac{\partial\gamma_{kl}}{\partial\partial_\mu\Phi^*} \propto \gamma^{kl}(\delta_k^\mu\partial_l\Phi+\delta_l^\mu\partial_k\Phi) \propto \partial^\mu\Phi\ . \tag{6}$$

This yields the field equation in the familiar form

$$\partial_\mu \left( \sqrt{-\det\gamma}\, \partial^\mu \Phi \right) = 0 \,. \tag{7}$$

The second field equation is just the complex conjugate. As usual, both can be combined in such way that the result would even hold if a mass term was present:

$$\begin{aligned} \Phi^* \partial_\mu \left( \sqrt{-\det\gamma}\, \partial^\mu \Phi \right) - \Phi \partial_\mu \left( \sqrt{-\det\gamma}\, \partial^\mu \Phi^* \right) = \\ \partial_\mu \left[ \sqrt{-\det\gamma}\, (\Phi^*\, \partial^\mu \Phi - \Phi \partial^\mu \Phi^*) \right] = 0 \,. \end{aligned} \tag{8}$$

# 3 Broken symmetry

The field equations reflect the symmetry of the line element (1). All the references are to the internal metric $\gamma$ rather than to the metric $g$ of spacetime, where the observers live. The field equations are nonlinear, but linearity yet is a very good approximation, hence the fields behave like waves. As observation teaches, all known fields oscillate around the same 4-submanifold that can be addressed as spacetime. The absolute values of the scalar fields have no tilt, their mean gradients relative to spacetime are zero as long as the densitiy of quanta is constant. What regards the phase, there is translation invariance left, but this does not change the situation. As an interpretation, the fields determine something that lives inside spacetime, namely currents of field quanta. This eventually means that spacetime is a preferred 4-dimensional frame of reference inside the 6-dimensional embedding space. Only if two different universes, each having its own position and orientation inside the embedding space (which has more than 6 dimensions if vectors are taken into account as well), collided, the symmetry would be relevant.

Since the frame is preferred, so is the stress-energy associated with it. The cosmological term is locked to our universe and its value cannot be changed by any coordinate transformation. *This broken symmetry explains why the vacuum does not gravitate* when the gravitational Lagrangian based on the curvature scalar is added. This is to be taken into account by subtracting the cosmological term from the action of the fields, only the higher order terms remaining. The action for a scalar field indexed $n$ relevant for gravitaton reads, much like Born and Infeld had proposed for vectors on a phenomenological basis

$$\mathscr{S}_{broken} = \frac{1}{\beta_n^2 G^2 \hbar} \int \left( -\sqrt{-\det \gamma_n} + \sqrt{-\det g} \right) \mathrm{d}^4x \approx \int \frac{1}{2} \hbar^2 \partial^\mu \Phi_n \partial_\mu \Phi_n^* \sqrt{-\det g} \, \mathrm{d}^4x \,. \qquad (9)$$

On the field equations (7, 8), this modification has no influence.

# 4 Relation to second quantization

The cosmological term is nothing but the vacuum energy density of the respective field, above quasi approached through the backdoor on a "classical" route. So far, only noninteracting fields are described, hence only "naked" values are involved. But apart from this, the value appearing in equation (4) should be in line with what is concluded from quantum physics of the respective field. Currently the tension is low, since noone knows where to exactly locate the frequency cutoff.

It was already discussed that the mechanism here is an escalation of the mechanism underlying Special Relativity, from the physics of objects living in time to the level of fields living in space-time. From this similarity originates one very strong indication that the above equivalence actually is realized. The role of the cosmological term here is comparable to the role of the rest mass term in Special Relativity. When a harmonic oscillator is treated relativistically in a fully consistent manner, this leads to a SO(2,1) symmetry, what directly progresses to the level of quantization: not the Heisenberg algebra is the quantum algebra of a relativistic oscillator, rather so(2,1) is. So, upon quantization, it turns out that mass simply is the ground state energy of the oscillator and is a quantum number in units of the energy quantum proportional to the frequency parameter [4, 5, 6] .

## 5 Origin of a mass term

While masslessness is obligatory for gauge fields, this is not so for scalars. In the case of symmetry breaking, the situation is even more subtle, in particular since the "mass" parameter has to be negative. But this shall not be regarded here.

There is a way to comprehend a mass term for any scalar field indexed $n$ on the basis of geometry. At any point in spacetime, perpendicular to it can be imagined a disc whose area is proportional to $\Phi_n^*\Phi_n$. When the area of the disk is integrated over spacetime, a 6-volume occurs. To allow addition to the 4-volume of the field manifold, a dimensionful coupling constant is to be introduced. Clearly, after most factors have cancelled out, eventually the mass is the coupling constant. It shall be written as $m_n^2 = \kappa_n^2 \frac{\hbar}{G}$, where $m_n$ is the mass, $\frac{\hbar}{G}$ is the Planck mass squared, and $\kappa_n$ is dimensionless. For the currently known masses not only of scalars, $\kappa << 1$. This can be supplemented to the action as usual

$$S_{mass\,n} = -\frac{\kappa_n^2 \hbar}{2G} \int \Phi_n^* \Phi_n \sqrt{-\det g}\, \mathrm{d}^4 x\,. \tag{10}$$

## 6 Conclusion

The idea of extra dimensions has been around for more than a century, without final conclusion. These extra dimensions have been given a number of interpretations in terms of physics, but none is so straightforward and simple as the one presented here. Special Relativity taught how seemingly completly different concepts eventually turn out as degrees of freedom in a common space equipped with a more or less complicated metric or comparable structure. The mechanism explaining where the derivative terms in the actions consequently come from is equally compelling in Special Relativity based on $c$ as in the relations presented here based on $G$, hence it could be fruitful to follow up on it. Scalars and vectors can be treated well, whereas gravitation and fermions require additional considerations.

The appearance of a cosmological term in this "classical" description offers the possibility to enter the vacuum quasi through the backdoor. The lessons learned from a harmonic oscillator in Special Relativity suggest that there are insights to be achieved that go beyond the current understanding of the quantum vacuum and its role for gravitation.

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