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Why Are There Laws of Nature?

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Abstract Resolving the enigma of information via a self-consistency requirement implies that the information content of nature is logarithmically small, and this is the origin for the observed laws of nature, to avoid information paradoxes.

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1 Introduction

Redundancies, also called symmetries, invariances, or laws of nature, are the subject of the physical sciences. They allow forward predicting parts of nature from how they look here and now. There are numerous interesting aspects of such invariances, and Noether's theorem is a key aspect. Things may be more subtle where no dimensional entity, such as space-time, has yet emerged. Quantum aspects further complicate the situation, while they make quite clear that *information* is at the core of everything. Indeed, the most basic questions about nature eventually drill down to the level where the only concept left is that of information itself: Why is nature not completely random? To what degree is nature redundant and what is the reason for such redundancies? How hard are these redundancies to recognize?

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With information so much at the center, in the following section I start the discussion with a revision of the current state of these concepts. In subsequent sections I apply the conclusions to an exemplary case and then to nature itself.

2 The enigma of information

A textbook on information theory starts with a reference to the Iron Age [1]: A blacksmith of that period may have been very skillful in handling iron, while having no idea of what iron actually was. One needs insight down to the level of atoms to comprehend the nature of different substances. Living in the Information Age, the book argues, we may be equally ignorant of what information is. Then the book continues with concepts whose core is the semantic content of some elements of information.

So the question is put: Do we know what information is? To find an answer, as an illustration, I use a television screen composed of N pixels. I call the element of entropy a pixel, and sometimes a bit or digit. The base of the exponential (logarithmic) function is not relevant for the conclusions, exp and log can be understood in an abstract sense. I use the abbreviation TV for the apparatus to avoid the notion of a screen, which suggests a specific meaning in current theoretical physics. It is important to clarify that the TV is regarded as extending over two spatial dimensions only, and specifically not over time. So the TV can show an image rather than a movie.

One can distinguish three levels of information theory:

1. Information theory just is the theory of entropy, with no distinction between the two concepts. This can be seen from the identity of the basic equations of information theory with those of statistical physics. The Shannon entropy is $S = -\sum_{w=1}^{W} p_w \log p_w$, where p_w is the probability of the system being in state w out of W possible different states. In the simplest case where all the p_w are equal, the Boltzmann entropy $S = \log W$ arises (Boltzmann's constant can be set unity). For the probabilities to become relevant there is an ensemble of screens in this case, usually indexed by time. In this sense, information theory is a well-founded, transparent and valuable tool for handling (streaming) bits.
2. Information theory addresses the interplay of possibility and actuality. Thus, there is an actual state w_{actual} with the probabilities collapsing towards the Kronecker delta, $p_w \rightarrow \delta_{w\ w_{actual}}$. Substituting the collapsed probability function into the Shannon formula, no entropy remains. Hence, entropy has been removed as information $\mathcal{I}$ and we may express this as $\mathcal{I} = S(\delta_{w\ w_{actual}}) - S(p_w) = -S(p_w)$, where the overall sign is a matter of choice.
   In our example, the TV can take on $W = \exp N$ different states with equal probability, so the entropy is $S = N$. However, the TV does not take on the superposition of all these possible states, rather it takes on one specific actual state. The collapse does not occur inside the TV, rather it takes place inside an observer whose ignorance is lifted. This sounds very strange, and

even stranger for classical physics than quantum physics, since a classical TV is never in any superposition of possibilities. In this sense, information is a rather strange concept related to a number of paradoxes.

3. Information theory deals with the semantic content. A well-functioning TV does not take on any actual state, rather it takes on a state showing something *sensible*. An observer undergoes not just an experience, but rather an *aha* experience. Semantic content is what [1] tries to formalize. However, semantic content is completely separate from the information concepts discussed above. The TV showing a Picasso or some typewritten tax declaration have in common that the TV is of equal size, i.e., equal N, and hence equal $\mathcal{I}$, what makes them indistinguishable. In fact, N is the last criterion regarded as essential with relation to semantic content. Thus, information theory is decoupled from the quantitative description through natural sciences and mathematics.

Thus, the answer to the introductory question is No: We do not know what information is, despite its omnipresence in modern physics, the entropy of black holes, information loss [2], entanglement [3], and many more areas. If the enigma of information is to be solved the conflict between possibility and actuality must be removed and the semantic content must be established as a mathematical object (i.e., a number) related to the entropy differently from $\mathcal{I}$.

3 A self-advertiser

Consider the following situation which occurs in everyday life. Entering a store for electronic equipment, you see a TV displaying a message such as “5 Megapixel TV – Promotion”. Addressed to humans, this contains some linguistic encoding and an additional semantic element. The fonts are human-sized as well, and consume a lot of pixels. On the other hand, 5 million is a quite special number with low information content, as is clear from its decadic representation. In the genuine case, however, the number of pixels needed to encode a number is the logarithm of that number, and the rest of the TV can be filled with redundant content. This yields the numbers characterizing the TV shown in Table 1, where I include an additional column referring to the corresponding thermodynamic quantities.

Hence, the enigma of information is resolved if and only if the semantic content of a message is the related number of degrees of freedom. Thus, as indicated in the table, information is the logarithm of entropy. This self-consistency shows non-fallacious information is logarithmically small compared to the degrees of freedom available. All the rest is redundant, i.e., fallacious information.

In quantitative terms, some further sharpening appears as feasible. The most straightforward implementation of redundancy is to repeat the same message several times. However, in that case the message need not encode the number of bits, it is sufficient if it encodes the number of repetitions, and this

Table 1 Numbers characterizing the TV described in the text, and the corresponding thermodynamic quantities

Quantity	Value	Thermodynamic correspondence
Number of pixels	N	Entropy
Number of possible states	$\exp N$	Phase Space Volume occupied
Number of actual states	1	
Semantic content of actual state	N	
Non-fallacious information	$\log N$	

completely separates information carrying elements from redundancies: All the replicas are exactly the same, and each -(incompressible) replica contains the entire information. If the number of replicas is R and the information is I, then $I = \log R$ and

$$N = S = R \cdot \log R \ = \ I \cdot \exp I \ . \tag{1}$$

There are subtleties in the details, but these do not disturb the overall concept. For the TV everything is in terms of natural numbers, with the number of pixels needed to encode a number actually being a step function. For the number 1, one bit is needed and the binary code is 1, despite that the logarithm of 1 is 0. The value 2 needs two bits with code 10, 3 also needs 2 bits with code 11, and so on, in the familiar binary representation.

4 Subsystems

Since information is the logarithm of entropy, subsystems must be reconsidered. We need to adapt the addition law for entropy to conform to eqn. (1). Let R_k be the number of replicas associated with subTV k, so that $R := \sum R_k$, where the sum runs over the entire ensemble; and $p_k := \frac{R_k}{R}$, be the probability that any given replica is associated with subTV k. As a mathematical identity, $R \log R = \sum R_k \log R_k - R \sum p_k \log p_k$, where the latter term is the Shannon entropy of R elements. The relation has the reasonable form $S = \sum S_k + S_{Shannon}$. Consequently for such an ensemble, the number of replicas is additive while the number of pixels acquires a correcting (Shannon) term.

The more subtle problem is the behavior of I. If all the subsystems stood independently, their information contents would add to $\sum I_k = \sum \log R_k$, hence $\exp \sum I_k = \prod R_k$. However, since R is additive rather than multiplicative, actually $\exp I = \sum R_k$. The product is greater than (or exceptionally equal to) the sum, except when the overwhelming majority of subTVs only represents 1 bit, but such case has vanishingly small information content. Thus, R_k cannot be independent quantities, rather R determines them even if it is way smaller than $\prod R_k$. Although this is quite a trivial insight, it clarifies that the division into subTVs actually means that the appearance of independence for R_k is fallacious.

5 Nature and quantum

To use the example of the TV to learn about nature as a whole, the following definition is used although it is close to circular logic: Nature is the entirety of what is relevant to be considered. It is the entity whose number of degrees of freedom is encoded as discussed. In an important departure from the TV example, time is included in the degrees of freedom determining the number of pixels of nature, N_{nature}. Nature spans all dimensions if the concept of dimensionality is relevant at all. There may be some parts of nature where it makes no sense to speak of dimensions and there may be some parts where more than one time-like dimension exists. For the part familiar to us, nature spans the three space-like dimensions plus time. So the state of nature is not dynamic, rather it is the entirety of what has ever been, is, and will be. As an example for a subsystem, a black hole comprises the entire history of the material that, for some duration of external time, is inside the hole, which first had fallen in and later will be irradiated off.

So, as a tautology, nature is in one and only one actual state. The inclusion of time also removes the difficulties caused by quantum aspects. Either there is a wave function whose evolution, which is deterministic, comprises the entirety of nature, or something must be added to the wave function to make it collapse. In the first case, the wave function can be encoded in terms of classical bits (C-bits), as any physicist does when expressing a wave function, whereas the second case requires further degrees of freedom, which can be encoded in terms of C-bits: These bits hold the outcome of the measurements, which are always C-numbers. It has been argued that beneath the quantum appearance, there is a mechanism similar to cellular automata [4], and that fits with the concepts presented here.

6 Information and illusion

Clearly, nature incorporates symmetries. These include symmetries in time, these may be the most important of all since they allow predicting the future. However, it is unknown how such redundancies arise, and what they mean in quantitative terms. Now a highly plausible answer may be taken from Table 1 with N replaced by N_{nature}.

How can the information content of nature be that small? Despite the recognizable symmetries, the major part of nature does not appear to be governed by laws, with black holes as perhaps the most impressive example. The material comprising a black hole appears completely arbitrary, and consequently Bekenstein-Hawking entropy seems to be necessary to encode it [5]. This leads to the black hole information paradox, since the hole only exhibits its macroscopic parameters. The discussion in the previous section shows a radical insight removing the paradox: Only the macroscopic parameters of the black hole are non-fallacious information, determining everything else. This is as well the case for a system underlying classical statistical physics, where experience argues that macroscopic parameters are sufficient to define the relevant aspects of the system. The insight is that these parameters not only encode the relevant aspects, they encode *all* of the aspects. Actually, such systems contain even less information as they correspond to subsystems of nature as discussed above, with mutually correlated macroscopic parameters.

This leads to our universe as another example for a subsystem of nature. It is questionable whether its estimated parameters are relevant for the sake of calculating the information. In particular, the contribution of dark components is unknown. Furthermore, in the light of recent developments, one could conjecture that the dominant contributions come from the cosmic horizon. If the mass visible inside the horizon is inserted, as a rough estimate one can still use the number given in [6] $N_{universe} \approx 10^{123} = 2^{409}$, which implies only a few hundred bits are required to encode all the non-fallacious information. Anyway, that the non-fallacious information of the universe is very small, can be seen from the highly ordered initial state. [6] shows a drawing showing the Creator pointing at a tiny portion of phase space, which illustrates well how small the initially occupied phase space volume was compared to the phase space volume available. This suggests that at the big bang (or big bounce or whatever) non-fallacious information was directly visible, while over time fallacious contributions entered. This fallacious information arrives camouflaged. Abstracted from the previously mentioned concept of cellular automata, I would like to continue the discussion without excluding any possibility.

In fact, there are numerous ways of filling the TV. One possibility is to repeat the number of pixels several times, another possibility is to make the background monocolored, etc. While these examples exhibit redundancy, there are much more subtle alternatives. It is a remarkably self-contradictory, although successful, exercise to write computer code generating a long sequence of apparently random numbers. The simplest example is a multiplicative linear congruential generator based on the mapping $x_n \to x_{n+1} = (ax_n + b) \bmod m$, with the constants carefully chosen. Fibonacci series generators are another frequent choice for practical use [7]. The essential parts of such a machine need a few hundred bits for encoding, while they can produce a sequence of pseudo-random numbers which is the exponential of some hundred bits long. So the pseudo-random sequences are exponentially longer than the necessary generating code.

If the code of the generator itself is interpreted as a number N (whose encoding needs $\log N$ pixels), then this machine produces order of magnitude N pseudo-random numbers. One can use this sequence to fill the entire TV with apparently random pixels. However, the corresponding information, of magnitude N, is fallacious. Something similar must occur in our universe, at core stands the code, which becomes more and more diluted by its own output.

7 Cardinality

For the TV, everything is finite, whereas for nature, the corresponding possibilities are shown in Table 2, where the number of degrees of freedom for nature is N_{nature}, $\beth_0$ is the cardinality of the natural numbers, and $\beth_1$ is the cardinality of the continuum. Neither of the first two variants can be realized, since the logarithm of $\beth_0$ does not exist. The first alternative is $N_{nature} = \beth_1$, and its logarithm is the cardinality of the natural numbers. Higher Beth numbers are against evidence, since quantum physics requires countability to be a characteristic of nature, in some form. As the second alternative, both N_{nature} and its logarithm are finite. In that case, N_{nature} is integer, and the logarithm is also integer by construction, since the following integer is taken.

Table 2 Variants for the values of N_{nature}

#	$\exp N_{nature}$	N_{nature}	$\log N_{nature}$	remark
1	$\beth_0$	not existent	not existent	impossible
2	$\beth_1$	$\beth_0$	not existent	impossible
3	$\beth_2$	$\beth_1$	$\beth_0$	eventually excluded
4	$\beth_a \quad a > 2$	$\beth_{a-1}$	$\beth_{a-2}$	against evidence
5	finite	finite	finite	integer

Variant 3 evokes the apparent subtle interplay between the classical continuous perception of nature and quantization. Nevertheless, it is very poorly plausible due to the scale invariance of the continuum: An arbitrarily tiny portion of the relevant space would contain an infinite amount of non-redundant information. Planck's constant supplements the symplectic structure of phase space known from classical theory with a unit of volume. Although quantum physics is not yet fully understood, there is ample evidence that the phase space volume is countable. So the cardinality of $\exp N_{nature}$ is at most $\beth_0$, and from the first row of Table 2 it follows that no infinity can occur at all. Furthermore, in current gravitational physics, elementary cells can be found for entropy, which exist in ordinary space. In particular, the 2-dimensional horizon of a black hole may be regarded as composed of elementary pieces with size of the Planck area. Bekenstein [8] relates this area to the associated entropy, with the elementary pieces of the horizon acting as bits. This implies countability for N_{nature} when gravitation is included as it always has to be, and now the second row of Table 2 implies that no infinity can occur. Hence, variant 5 is the only survivor, and I conclude that N_{nature} is finite, which completes the analogy with the TV.

8 Conclusion

The concrete form of the laws of nature were not discussed here, rather the focus was on the amount of redundancy. Despite this, and any further not yet understood aspects, the following conclusions can be drawn:

- The laws of nature originate from a self-consistency requirement for information.
- Non-fallacious information is logarithmically small, everything else is governed by laws (redundancies).
- Apart from the evident Noether currents, the redundancies are camouflaged.
- In terms of the non-fallacious information, there are no information paradoxes.
- Nature is finite.

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