A Symmetry Involving Newton's Constant as Basis for the Born - Infeld Theory

Gerald Vones

Graz, Austria, European Union
gerald@vones.eu
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Abstract

This paper argues that there is a symmetry mediated by Newton's constant based on an 8-dimensional space that can be considered as space-time-energy-momentum. Its metric includes a skew-symmetric part. The resulting relations, if specified to the electromagnetic field are those of the initial Born-Infeld theory, however with the scale factor being determined by the Planck mass rather than by the electron mass. Some implications, including the role of the vacuum energy density as well as the topology of spacetime are discussed.

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1 Introduction

The most fundamental symmetries of nature are those mediated by the elements of the Planckian set of units: the velocity of light c, Planck's (reduced) constant $\hbar$ and Newton's constant G. Because of its very transparent role, c has become the star among the natural constants, and there is no need for further commenting. Since it is nothing but a representation of the pure number 1, in theoretical articles c usually is set unity explicitly, and this will be followed here.

The case of $\hbar$ is a little more sophisticated. Noone ultimately comprehends quantum physics, but one can well conclude that the phase space volume is to be converted into pure numbers in a uniquely defined way by means of the volume form $\mathrm{d}\Theta = \frac{\mathrm{d}x \wedge \mathrm{d}p}{\hbar}$, where x is position (of an object in 3-space) and p is associated momentum. For second quantization, in place of x and p come fields and their associated momenta. This all works sufficiently well except for gravitation. It can be added that $\hbar$ oftenly is regarded as converting length into inverse momentum by virtue of the quantum condition $\mathrm{d}\Theta = 1$. But this is, at best, half of the story. To reach full equivalence between physical and mathematical objects a further conversion factor is needed.

G appears in the Einstein - Hilbert action $\frac{1}{2G}\int_{spacetime} R\,\mathrm{d}V_4$, where R is the curvature scalar and $\mathrm{d}V_4$ is the 4-volume element of the manifold indicated (spacetime in this case). Since G converts R with dimension $length^{-2}$ into an energy density, this is substantially less transparent than the roles of c and $\hbar$. But, as an element of the Planckian set of units, G must act in a universal way. In fact, the Schwarzschild radius is $2G$ times the mass. Despite of this relation only being a well camouflaged hint towards a deeper symmetry, it suggests

Conjecture 1: $2G$ is the unambigous universal conversion factor between quantities with the dimension of length (including time) and such with the dimension of momentum (including energy).

What regards the other obligation from being a member of the Planckian set, namely that the mediated symmetry has to be simple like for c and $\hbar$, gravity as currently formulated obviously is not where this is realized. General Relativity as any physical theory is based on a symmetry, the conservation of the stress-energy tensor. However, that the density of energymomentum (alias stress-energy) is a source for a property of spacetime, namely the metric, connects these two concepts in a way that hardly could be more asymetric. The dimension of G being $\frac{length}{momentum}$, whereas those of c is $\frac{length}{time}$, suggests some generalized metric as the appropriate ansatz. [1] discuss a (1+4)-dimensional Minkowski space in their equation

(5). But it is difficult to comprehend why this fourth spacelike degree of freedom should differ from the other three by (just) its extremal length scale in our everyday units. Rather, momentum (energy, mass) should be something qualitatively different from space. c mediates an indefinite metric even surpassing the ingenuity of mathematicians at the time when Special Relativity was introduced - so there is a 4-dimensional spacetime, nevertheless time is qualitiatively different from space. The next such escalation is a metric containing a skew-symmetric part in addition to a symmetric (Riemannian) part. Such a skew-symmetric part furthermore is strongly suggested by the well known symplectic structure of phase space. This leads to

Conjecture 2: $2G$ appears in a generalized metric containing a skew-symmetric part in a context which is *not* gravitation, notwithstanding the need for unification at a fully developed stage. It reads for one spacetime-like x plus one energymomentum-like p making up the position vector $\mathrm{col}(x, p)$

$$\daleth_1 = \begin{pmatrix} k_1 & 2G \\ -2G & 4G^2 k_2 \end{pmatrix} , \tag{1}$$

where k_1, k_2 are parameters wich can take values either 1, 0 or 0, 1 or 1, 1 (the possibility of complex values of these constants shall not be regarded).

Obviously, at least one k_i must be unequal zero. If both are unequal zero, their signs must be equal, else the metric determinant would vanish. Without loss of generality, this sign can be chosen positive: first, the overall sign of the metric is irrelevant; second, the relative sign compared to the skew-symmetric term is irrelevant, since this corresponds to the two possible signs of motion (or, as will turn out here, charges) which can have no influence on the results.

In this paper, the two conjectures shall be applied to a specific case. The new symmetry will turn out to imply a theory of nongravitational vector force fields brought forward by Born and Infeld in 1934 [2] if $k_i = 1, 0$ are set, apart from the value of their scale parameter. For clarity it can be added that those reference has unusual sign conventions (see in particular their equations 2.8, 3.6 und 4.11). Reference [3] is of better readability in this respect.

2 An elementary symmetry involving G

An elemetary charge q aquires in the respective force vector field A an additional momentum 4-vector $P = qA$. The value of the coupling constant, thus the charge, is to be understood as the low-energy limit. P (not the full momentum), *abstracted* *from the concrecte force field as well as from any carrier of the charge*, can be combined with spacetime in a manner suggested above, namely to an 8-dimensional space (position vector denoted as $\mathrm{col}(x^0, P^0, x^1, P^1, ...)$) with a generalized metric which is the tensor product

$$\daleth = g \otimes \begin{pmatrix} 1 & 2G \\ -2G & 0 \end{pmatrix} , \tag{2}$$

where g is the usual Riemannian metric of spacetime. $k_2 = 0$ is forced by the helicity of the vector fields, whereas a suppression of this term by a factor $4G^2$ would not be sufficient. This will become more transparent in the next section. So this generalized metric is the sum of a degenerate Riemannian part (its determinant vanishes) plus a skew-symmetric part. In toto, the metric is non-degenerate. The way g enters has not to do with the considerations here, rather follows the ansatz of [2].

Equipped with this metric, *there is one single 8-dimensional "space-time-energy-momentum" (8-stem in the following) spanned by spacetime and P*. Any specific vector field (multiplied by the respective elementary charge) - indexed by a - is characterized by its specific field equations $P_a^\mu(x^\nu)$. These 4 equations define the embedding of a respective 4-dimensional "field manifold" $\mathscr{M}_a$ in one and only 8-stem - like there are many particle worldlines embedded in one and only spacetime in particle physics. Up to now 12 force fields are known to exist, and their field manifolds can be assigned $a = 1$ to weak hypercharge, $a = 2, 3, 4$ to weak isospin and $a = 5, ..., 12$ to the gluons.

In the following, the dynmamics of the $\mathscr{M}_a$ will be discussed. The field equations will turn out as nonlinear. It can be clarified that current field theories except for electromagnetism are nonlinear, but as a consequence of internal symmetries what is a completly different aspect which shall not be regarded here.

3 The action for the fields

In current field physics, the field strength tensor $F^{\mu\nu}$ is defined as the anti-symmetric gradient of the respective vector field. The Lagrangian density proportional to $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ strongly resembles the nonrelativistic $\frac{1}{2}\vec{p}^2$ from particle physics, and is equally implausible. The modification to be performed is exactly the same as was done from nonrelativistic to relativistic physics, where the Lagrangian was geometrized as the 1-volume of the worldline: *the Lagrangian is to be geometrized as the 4-volume of the respective* $\mathscr{M}_a$.

To arrive at the 4-volume of any $\mathscr{M}_a$, the induced metric on it is to be calculated by virtue of Gauss' formula $\gamma_{a\mu\nu} = \daleth_{mn}\frac{\mathrm{D}X_a^m}{\mathrm{D}x^\mu}\frac{\mathrm{D}X_a^n}{\mathrm{D}x^\nu}$ where $X_a = (x, P_a)$ are the eight contravariant embedding space coordinates at the location of $\mathscr{M}_a$, and x are the four coordinates on the respective $\mathscr{M}_a$ simply identified with part of the X. Since 8-stem is intrinsically curved, D are covariant derivatives w.r.t. the coordinates used on $\mathscr{M}_a$, i.e. the x^μ. ℸ ist to be taken from equation (2). With this, the Gaussian formula yields after a short calculation

$$\gamma_{a\,\mu\nu} = g_{\mu\nu} + 2G\tilde{F}_{a\,\mu\nu} \ , \tag{3}$$

where (index a supressed) $\tilde{F}_{\mu\nu} = P_{\mu,\nu} - P_{\nu,\mu}$, the comma indicating partial derivative w.r.t. the coordinate following. Mind that this is in terms of P, consequently when moving to the untilted quantities there comes the respective elementary charge as a further factor. Only ordinary (shall mean free of gravitational connections) derivatives are involved for any g because of the antisymmetry of $\tilde{F}$ in union with the symmetry of the gravitational Christoffel symbols.

Hence, the action for the field indexed by a is (for the second line of the equations, see reference [3])

$$\begin{aligned} S = const \cdot \int_{\mathscr{M}_a} \mathrm{d}V_4 = const \cdot \int \sqrt{-\det \gamma_{a\,\mu\nu}}\ \mathrm{d}^4x = \\ = -\frac{1}{4q_a^2G^2} \int \sqrt{1 + 2q_a^2G^2F_{a\,\mu\nu}F_a^{\mu\nu} + \ldots\ldots}\ \sqrt{-\det g_{\mu\nu}}\ \mathrm{d}^4x \ , \end{aligned} \tag{4}$$

where q_a is the respective elementary charge. Of course, all this is at the level of massless fields prior to any symmetry breaking.

4 Relation to the Born - Infeld theory

Comparing the above equation to equation (12.5.1) of reference [3], one can read off the new value assigned to the Born - Infeld parameter called b there and in the original literature (b^2 is the reference scale for the energy density, which actually is the vacuum energy density of the respective vector field) as

$$b \rightarrow \frac{1}{2q_aG} \ . \tag{5}$$

In the historic example case of electrodynamics, which nowadays is known to be the only massless survivor of the Higgs mechanism related to the electroweak force in a tricky way, g_a is the electromagnetic elementary charge e.

Born and Infeld almost formulated such theory for electrodynamics long ago, but they did not reckognize the fundamental symmetry behind. Their conjectured scale was the electron mass m_-, they had $b \approx 1,24 \cdot \frac{m_-^2}{e^3}$. This was a quite straightforward idea in those days, but is wrong for sure given the current knowledge.

Facing the particle zoo, it is quite clear that not any of the particle masses can be the appropriate scale. Only the universal Planck mass is plausible. Equation (5) approximately puts approximately $\frac{1}{18}$ of the Planck mass in place of m_- compared to the value from [2] if $137 \cdot e^2 \approx \hbar$ is used. Because of lack of immediate practical relevance, the work of Born and Infeld has not been part of the main stream, but has not been ruled out either. A recent paper on this topic is [4]. Even so called Born - Infeld gravity theories are considered, for example by [5].

The U(1) invariance is woven into equation (2), and this makes sense. Namely, this invariance has directly to do with the spacetime properties of any vector field, projecting out the pure helicity 1 component (this would not work if the lower right entrance of the matrix in equation (2) was unequal zero). Internal symmetries other than U(1) could be added uncomprehendingly by replacing the appearing derivatives by such covariant w.r.t. the respective gauge group. This means in effect that the respective $\mathscr{M}_a$s mutually interact, for example all those associated with the gluons. However, whereas this is quite straightforward, actually this only covers part of the Lagrangian. As the standard model teaches, the full theory is much more sophisticated, especially in the sector where the Higgs mechanism is relevant. This holds even if only the hypercharge field is treated in accordance with the Born - Infeld theory, see [6]. But all this does not affect the fundamental insight referring to *the relation between spacetime and energymomentum and the role of G, which is the only topic of this paper*.

5 Vacuum energy densities and and their "unnatural" cancellation

Moving to the Hamiltonian density, the expansion of the root in equation (4) yields positive vacuum energy terms even larger than the Planckian density ρ_{pl} of one Planck mass per Planck length cubed. This was demonstrated for the exemplaric case of electromagnetism where $\frac{1}{4e^2G^2} \approx 34\,\rho_{pl}$. It is known that second quantization of the fields leads to vacuum terms of comparable size. *To make sense, these two aspects must be exactly one and the same.* This sounds elusive first, but there are strong arguments that this is actually the case.

A naive application of the ideas of second quantization would say that the vacuum energy density is determined by the ultraviolet cutoff of the else divergent momentum integral $\int^{k_{max}} k^3 \mathrm{d}k$. Since for the example case of electrodynamics it was shown that the vacuum energy density is proportional to $\frac{1}{e^2}$, the cutoff is determined by the inverse coupling constant $k_{max} \propto \frac{1}{\sqrt[4]{\alpha}}$. But the situation is more subtle here. The expression for the vacuum energy density exactly corresponds to rest mass in

Special Relativity (compare the second line in equation (4) to $-mc^2\sqrt{1-\left(\frac{v}{c}\right)^2}$, where m is mass and v is velocity), and an explanation must take this into account. As a further obstacle, the usual decomposition of the field into partial waves is limited by the nonlinearity of the theory. Whereas the correct values of the vacuum energy densities are to be estimated from the Hamiltonian density derived from equation (4), the following is an approximation, though a good one because of the smallness of the nonlinearities.

When harmonic oscillators are treated relativistically in a consistent way, this leads to a SO(2,1) symmetry. There is literature [7, 8, 9], here only the very conclusion shall be recapitulated. Rest mass is nothing but the ground state energy of the oscillator. In terms of the quantum $\hbar\omega$, (a caveat: ω is not the frequency of the oscillation, rather those is inversly proportional to the energy) the positive branch of the energy spectrum is $N+\frac{1}{2}, N+\frac{1}{2}+1, N+\frac{1}{2}+2, \ldots\ldots..$, where N is a nonnegative integer determining the mass. For second quantization, in anology, for any partial wave of the (massless) field with frequency ω, there are $N+\frac{1}{2}$ quanta in the vacuum, with the appropriate $N \in \mathbb{N}_0$. The frequency cutoff, in contrast, does not hold information on any individual vacuum energy density, rather is a universal constant with physical relevance.

Since observation teaches that vacuum energy densities do not gravitate, [2, 3] subtracted them without further explanation. The theory presented here gives a justification for this subtraction. This again is based on the anology to Special Relativity. The rest energy of, say, a bunch of electrons well exists, though can never be converted to energy of photons as long as there are no positrons around to allow for pair annihilation. Here, in anology, the vacuum stress-energy of the fields cannot be balanced by gravitational stress-energy (be it graspable or ungraspable like in General Relativity), except in processes that might have occured at the big bang. "Anti" field manifolds can in fact exist, since mathematically the spectrum of SO(2,1) has a negative branch as well, which is an exact mirror of the positive branch.

The dynamic objects are the $\mathscr{M}_a$. Yet one important 4-dimensional manifold is pending, namely 4-dimensional spacetime as such. It can be regarded as another submanifold of 8-stem, but it is not derived from variation w.r.t. the $P^{\mu}_{spacetime}$ - our spacetime is not an extremal manifold. Rather, the condition is that the induced metric on spacetime is $\gamma_{spacetime} \equiv g$, hence $\tilde{F}_{spacetime} \equiv 0$ in all indices. As the general solution, $P_{spacetime}$ has to be a pure divergence. The appropriate choice is $P^{\mu}_{spacetime}(x^{\nu}) = const^{\mu}$, and all these constants can be set zero without loss of generality. The kinetic (non-rest) energy of a particle is observer dependent, so its unambigous definition requires a preferred frame. In the context here, a preferred frame is given due to the orientation of spacetime as a submanifold of 8-stem.

Here is no analogy to Special Relativity, where an observer can live on just another extremal worldline. In contrast, we the observers live in spacetime which is not an extremal field manifold, and thus is preferred. This is reflected by the degenerate Riemannian part of the embedding space metric, where only spacetime counts. Gravitation, as described by General Relativity, is associated with this frame. The corresponding subtraction of the vacuum energy densites of course is exact by its very construction. So the miracle why such process is of seemingly unnatural precision is none.

There is a possibility of restoring the full symmetry of the embedding space, namely when a whole bunch of universes is regarded. Their different 4-dimensional spacetimes, together with their associated respective field manifolds, all live in one and only 8-stem.

6 Topology

What has been presented so far is a local picture of the situation. Aiming at the universal perspective, new facettes have to be taken into account. An important condition is that the fields have to be square integrable. So one is not allowed in 8-stem to rotate the three spacelike x^n, $n = 1,2,3$ into the respective P^n to arrive at a constant gradient of the fields. As said, there is a preferred frame.

There is a logic solution to the problem of constant spatial gradient, namely those of a torus topology of the configuration. Then the criterion of single-valuedness of the field manifolds guarantees that the mean spatial gradients of the fields vanish. At first, the cartesian structure of x^0 and P^0 shall be retained. But for any of the 3 spacelike pairs, x^n becomes an angle variable, whereas P^n becomes a radial degree of freedom. So the 4-dimensional spacetime has topology $R^+ \times T_3$, and so have all the field manifolds. g is a function of x^0 and the three angles.

As a metric manifold, spacetime is the cylinder $P^0_{spacetime}(x^\mu) = \text{const} := \mathscr{E}$, $P^n_{spacetime}(x^\mu) = \text{const} := \mathscr{R}$ with at this stage undetermined positive values for $\mathscr{E}$ and $\mathscr{R}$. Spatial isotropy was assumed in the sense possible (a torus as a metric manifold is not isotropic, nevertheless the three orthogonal spatial directions can be treated as equal). Again, $\gamma_{univ} \equiv g$ in any case, since only ordinary derivatives are involved in formula (3). The observed expansion of the universe over time is encoded in g, not in the $P^\mu_{spacetime}$. So, metrically this cylinder is intrinsically curved by virtue of g. Topologically, it suggests itself quite strongly to be modified to a 4-torus, with time another angle variable and P^0 another radial variable. Whether time actually is cyclic is unknown, but there is a theory claiming this [10].

7 Conclusion and Outlook

In this note, the historic Born - Infeld theory removing the linearity of electrodynamics is given a new basis. The transparent symmetry can be regarded as transferring the essence of Special Relativity to nongravitational field physics. The role played by c in Special Relativity is played by G here, which thus appears in a simple and clear relation - with some pecularity, though -, as can be expected for an element of the Planckian set of units.

It has been argued that pyhsics has gone astray in search for beauty [11], but this is a difficult statement insofar as beauty is a matter of taste. It appears that physicists rather search for symmetries, since physical laws are nothing but symmetries of nature as Noether brilliantly taught. For example, despite of lacking evidence for supersymmetry from the LHC it remains highly unlikely that bosons and fermions just live alongside each other unconnetedly. Have physicists overdone? Not at all, can well be argued, even the opposite. Apart from rare exceptions like reference [1], relations kind of equation (1) have not been conjectured despite of the historical paragon of Special Relativity and the obvious similarities between field actions proportional to $F^{\mu\nu}F_{\mu\nu}$ (be there internal symmetries or not) and nonrelativistic kinetic terms in particle physics.

It could be relevant that the Born - Infeld action plays a role in string theory as well. The mechanism presented in this paper yields a simple variant of a D-brane action [12] with no dilaton and the $\tilde{F}$ directly produced from the embedding (denoted B_{ab} in the reference equation 8.7.2). The associated string tension is $\frac{1}{2q_aG}$. Where ever this may lead to, it is noteworthy that the presented mechanism gives a new interpretation to known objects. A field manifold is kind of a D-brane, but lives in 8-stem. So despite of essentially the same mathematics, the physical consequences are completly different. This is like the difference between a spacelike thread (string would be a more appropriate notion) and a timelike worldline of an object.

What is covered here are the helicity-1 force fields. Since these are vector fields, they can be added to the position vector of spacetime. Importantly, $P = qA$ actually is (part of) a momentum, though of charged probes rather than of the fields under consideration. Whereas this is only a segment of the fields known, it is a central one. One among the millenium problems deals with Yang - Mills fields. The problem aims at the quantum behaviour, but clearly it plays a role what the classical action looks like.

As a drop of bitterness, direct observation of the central predictions, in particular the potential of a point charge, is hardly possible. At the Planckian scales, second quantiziation effects become dominant and blur the effects at the level of first quanitzation. A 3-torus topology of the universe, on the other hand, is not sufficient evidence for the theory presented. Eventually, if the ideas presented in this paper are correct they will have to appear everywhere in a theoretical framework comprising all fields. Unfortunately, the inclusion of other fields than vectors - even scalars - is not straightforward, despite of their currently used actions being equally implausible.

The most urgent problem in this respect is gravitation. In the above, the coupling of all the other force fields to gravitation is achieved by the pre-curvature of the embedding space, what fits perfectly to the ideas of General Relativity. That Pauli was unhappy with this path implicitly taken by [2] and pleaded for alternatives, is reported in [13]. But up to now no such alternatives have been developed and actually it is hard to imagine what they could be alike. As [2] remarked, it is even easy to incorparate the dynamics of the gravitational field itself simply by adding the Einstein - Hilbert Lagrangian. However, facing the new symmetry based on G presented in this paper, such pragmatism is far too little. Whether this symmetry can be brought in line with the mentioned Born - Infeld gravity theories [5] shall be the discussed elsewhere.

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