# Newton's Constant inside a Fundamental Symmetry

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**Abstract**

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## 1 Preliminaries

In the following I use units where the velocity of light $c$ in flat spacetime is unity (except where written out explicitely), while I retain Newton's constant $G$ and Planck's constant $h \equiv 2\pi\hbar$ as dimensionful. So the dimension of G is $\frac{length}{energy}$ or equivalent, and those of $\hbar$ is $length \cdot energy$ or equivalent.

A bosonic field $\phi$ naturally has dimension $\left(energy \cdot length^3\right)^{-\frac{1}{2}}$. This is because $m^2\phi^*\phi$, where $m$ is the mass, contributes to the energy density. In the context of field theory, fields with this natural dimension usually are multiplied by $\sqrt{\hbar^3}$, what eventually assigns to bosonic fields the dimension of $energy$.

For the electromagnetic field, the historic convention has been different since in Maxwell's equations Planck's constant does not appear because of the masslessness. The historic dimension for the electromagnetic vector field is $\sqrt{\frac{energy}{length}}$, so that its gradient has dimension $\sqrt{\frac{energy}{length^3}}$. The dimension of the electromagnetic vector field multiplied by the elementary charge $e$ is $energy$.

The metric felt in spacetime will be denoted $g$, while metrices induced on embedded manifolds via the Gauss formula will be denoted $\gamma$. Both are pure numbers. The tensor indices of the metrices usually are supressed in the notation, while the determinant symbol always is written out explicitely.

Planck length and Planck mass are $\ell_{pl} \equiv \sqrt{G\hbar}$ and $m_{pl} \equiv \sqrt{\frac{\hbar}{G}}$, repectively. In many cases the meaning of symbols will not be stated explicitely, since the notation is standard like $x$ for position and so on.

## 2 Fundamental Symmetries

It is commonplace that our current picture of nature is incomplete and even faulty in some respect. Most of all, the incompatibilty of General Relativity and Quantum Physics leaves us clueless, making obvious that the necessary modifications go down to the very roots. On the other hand, the current framework has accurately passed all tests from observation so far. If one aims at modifying the fundamentals, then one must come along with a quite convincing line of reasoning to motivate anybody to give this a try.

Ideas which have particularily been on the table for some decades are supersymmetry and string theory. In these and further disciplines, the skills have climbed to an incredible level and remarkable facts like duality have been discovered. It is very unlikely that the instruments developed yet are not sufficient. Rather, it could be a question of the very sight on nature. As an example, Special Relativity is the theory of timelike threads (a much better notion would be that of strings, but I do not use it for obvious reasons) called worldlines. The most skillful treatment of threads in space would not help to unveal the not so difficult truth.

As I shall argue there is a yawning gap, namely the fundamental symmetry Newton's constant *does not* mediate. At a first glance, such argumentation sounds dumb. The notion of "fundamental symmetry" is fuzzy and appears as correspondingly useless, while any physical equation - those of General Relativity in particular - expresses a symmetry. But despite of mathematics being the language of nature, doing physics is not the same as doing mathematics. Physics is about dimensionful concepts like length or mass and their interrelations. Dimensional analysis has been a powerful tool in many cases, while it means nothing to mathematics. Mathematics only knows pure numbers and there are no restrictions on their interrelations. What regards the pending fundamental symmetry, it is as with other things of fundamental character in life: you reckognize them on encounter. In fact, when regarding the Planckian set of units, two of them teach impressively what the third yet has failed to deliver. At its fundamental level, physics deals with 4 concepts: space, time, mass and pure numbers. Planck conjectured one century ago that the set $c, \hbar, G$ is the preferred set of constants to convert these 4 concepts into each other.

Special Relativity teaches that space and time are the same in the quintessence. Well, actually things are a bit subtle, and it needs expertise to appreciate and deal with all the implications related to the line element squared $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2$. Nevertheless, the deepest sense behind can be felt by any person of some minimum intelligence, since the underlying physical concepts are immediately and intuitively experienced by any human.

Statistical physics in union with quantization teaches that physical quantities and pure numbers are the same in the quintessence. It would be a misbelief that one can freely choose the conversion factor if one wants to do it correctly at the level of entropy alias information. It again needs expertise to comprehend the subtleties related to the volume form $\mathrm{d}\Theta = \frac{\mathrm{d}x \wedge \mathrm{d}p}{\hbar}$. Even more intriguing, this formula gives a "classical" picture, while there has a debate gone on for a century on how to interprete quantization [1]. But irrespective of the confusion at the experts level, the deepest sense behind docks to the ancient concepts and can be felt even by laypersons.

The symmetry mediated by $G$ in General Relativity looks quite different. First it deals with elements beyond the basic concepts, and second there is utmost asymetry between energymomentum and spacetime: the density of the one is a source for a property of the other. Well, from black hole theory emerged a formula saying that the horizon area and entropy are the same in the essence. This strongly supports Planck's conjecture about the role of his set of units. But the horizon of a black hole is something very distant to our everyday experience. Rather, learning from $c$ and $\hbar$, the symmetry mediated by $G$ should read *spacetime and energymomentum are the same in the quintessence.* In the following, I shall discuss what this can mean at some level of expertise.

## 3 Hermitean Spacetimeenergymomentum

Spacetime can be characterized by a position vector. So, if energymomentum shall be the same in the essence, it must enter as a vector rather than a tensor density. One can find the appropriate candidate from the minimal electromagnetic coupling $p - eA$. It can be clarified that I deal here with the spacetime aspects alone. So the phenomena of nonabelian fields emerging from their internal symmetries (plus even more tricky aspects from the Higgs mechanism) are not covered, and the discussion can be restricted to the electromagnetic field. Whether or not the elementary charge is that of an electron or $\frac{1}{3}$ of it shall be left open.

Conclusively, energymomentum is those aquired by an elementary charge in the electromagnetic field and it has to appear in a symplectic way together with $x$, like $p$ does. From this I conjecture the hermitean line element

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu \pm \imath\, G\, \mathrm{d}x_\mu \wedge \mathrm{d}\, eA^\mu \,, \tag{1}$$

where the indices are raised and lowered by means of the metric of spacetime $g$ which can correspond to intrinsic curvature in general. Hence, the metric $\Gamma$ in the 8-dimensional space spanned by $x$ and $eA$ is the tensor product

$$\Gamma = g \otimes \begin{pmatrix} 1 & \mp\imath G \\ \pm\imath G & 0 \end{pmatrix} \,. \tag{2}$$

The second factor is a promising candidate for a fundamental symmetry involving $G$.

The 4 field equations $A^\nu(x^\mu)$ define a 4-dimensional submanifold $\mathcal{M}$ - a brane, so to say - of the 8-dimensional hermitean space. If one extremizes the 4-volume of $\mathcal{M}$, one comes very close to a theory first formulated in the 1930s by Born and Infeld. The metric induced on $\mathcal{M}$ can be computed from Gauss' formula $\gamma_{\mu\nu} = \Gamma_{ab} \frac{\partial X^a}{\partial x^\mu} \frac{\partial X^b}{\partial x^\nu}$. The latin indices run over the 8 embedding space degrees of freedom, while for the coordinates on $\mathcal{M}$, I used the 4 spacetime degrees of freedom. The result is

$$\gamma_{\mu\nu} = g_{\mu\nu} \pm \imath e G F_{\mu\nu} \ , \tag{3}$$

where $F_{\mu\nu} = \mathrm{D}_\mu A_\nu - \mathrm{D}_\nu A_\mu$ is the field tensor. The derivatives $D$ are automatically covariant w.r.t. $g$, since everything is properly formulated in terms of geometry. If there is no torsion of spacetime, as shall be assumed in resemblance to General Relativity here and lateron, only the ordinary derivatives remain.

One can compute the determinant of this metric [2, 3] and arrives at $-\det\gamma = (-\det g)\ \left[1 - \frac{e^2G^2F^2}{2} - \frac{(e^2G^2FF^*)^2}{16}\right]$, where $F^*$ is the Hodge dual of $F$. So, to regain the correct limit, the Lagrangian to be integrated over the spacetime 4-volume $\sqrt{-\det g}\ \mathrm{d}^4x$ reads:

$$L = \frac{1}{e^2G^2}\frac{\sqrt{-\det\gamma}}{\sqrt{-\det g}} \approx \frac{1}{e^2G^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{\rho}{\alpha} - \frac{1}{4}g_{\alpha\mu}g_{\beta\nu}F^{\alpha\beta}F^{\mu\nu} \ , \tag{4}$$

where $\rho = \frac{1}{\hbar G^2}$ ist the Planckian energy density, that is one Planck mass per Planck length cubed, and $\alpha$ is the fine structure constant.

## 4 The Born-Infeld Theory

Born and Infeld [3] essentially started from the Lagrangian (4). However, there are some very remarkable aspects about their paper: Firstly, they did not write out the decisive relation (1) - and to my knowledge noone yet has done so. Secondly, they did not involve the imaginary unit. Thirdly, they used the electron mass as the relevant scale rather than the Planck mass. Fourthly, they substracted the vacuum energy density.

Most of these facts can be well understood in the historic context. It was a philosophical idea that spacetime, equipped with a metric structure, in some sense is all that exists. This inhibited the comprehension of the role of $G$ as presented here. Since there was no respective lineelement, there was no basis for considering its hermeticity either. As a result, their factor in front of $F^2$ had the same sign as the vacuum energy density. The problem of the electron radius was the main target of the Born-Infeld theory and it was quite straightforward in those times to take the electron mass as the scale. Today this is beyond any plausibility and the only serious candidate is the Planck mass, implying a universal symmetry. The vacuum energy density was removed because it does not gravitate. But from the above considerations it is quite clear that the vacuum energy density enjoys an equally factual existence as the rest mass does in Special Relativity. That it does not gravitate must emerge from the classical gravitational equations equally automatically as the fact of electrodynamics that an electron cannot radiate away its rest mass except in pair annihilation.

## 5 The Gravitational Field

As said, nonabelian gauge fields are not a matter of consideration here. But the gravitational field is, in particular since $G$ is the gravitational constant. However, gravitation remains tricky as it has always been.

The way it involves the metric coupling of the electromagnetic field appears as one of the strongest features of the Born-Infeld theory. As analysed above, the line element (2) straightforwardly contains the spacetime metric, what guarantees that gravitational couplings are metric as it is so strongly supported from observation. On the other hand, this is a pre-curvature of the embedding space. Why so? As [4] reports, Pauli was unhappy with the Born-Infeld gravitational coupling and proposed an alternative, which however is quite contrary to the path followed here.

This leads to the question about the vacuum and the large-scale shape of the universe. While $\gamma$ can so clearly be understood from the Gaussian embedding mechanism, the metric of spacetime seems to have a different origin. Despite of the metric being so much at the center, spacetime is not an extremal manifold - or is it? To me personally it appears as most plausible that the universe is an extremal manifold to a very good approximation, like any planet approximately is a sphere (flattened from rotation) despite of the weird and different detailed surface geometries. But if this was true, what about pre-curvature of the embedding space?

My best current guess is: the embedding space is flat, and classical gravitation has to be described by teleparallelism or some related concept; but the interpretation of the gravitational manifold $\mathcal{M}_{grav}$ needs further consideration.

In the following, I go for the most straightforward modification of the above concept for the case of gravity. The main difference to the nongravitational fields is that for gravity the 4-volume of $\mathcal{M}_{grav}$ is null (its metric determinant vanishes), like the line action of a photon is null in particle physics. So the line element (2) remains as it is. A second specifity of the gravitational vector field is that it generates the spacetime metric $g$. This is called an indirectly coupling gravitational field. Such a mechanism was already proposed by [5] in the early 1070s. But he used a scalar as the primary gravitational field, and the consequent lack of local orientations made those theory fail.

In the following, $A$ and $F$ are the gravitational field. I supress indicem $_{grav}$ for the sake of readability. But this index is present everywhere, not only for the fields but in particular for $\alpha$. So the gravitational Lagrangian reads

$$L_{grav} = \lambda \left[ \frac{\rho}{\alpha} - \frac{F^2}{2} - \frac{\alpha (FF^*)^2}{16\rho} \right] := \lambda\, U \ , \tag{5}$$

where $\alpha$ is the so long unknown dimensionless coupling constant of gravity. The Langrange multiplier field $\lambda$ is a pure number. As source term, one could in particular take the Born-Infeld Lagriangan for the electromagenetic field. But I simply take a cosmological term plus the ordinary matter + radiation Lagrangian, since the latter is not the focus here. So the Lagrangian to be integrated over $\mathrm{d}^4x$ (indicated by a hat over the symbol) is

$$\hat{L} = \lambda\, U + \sqrt{-\det g}\ (\frac{\Lambda}{G} + L_{ordinary}) \ , \tag{6}$$

where $\Lambda$ is a constant. This constant is huge, but as a first observation there is no stiff connection to the Lagrangian of the gravitational field, since the field $\lambda$ is involved.

As said, the gravitational field produces the spacetime metric. As a consequence, the action is not invariant under the U(1) gauge transformation, what is highly plausible for a description of gravitation. The proposed mechanism is: $A$ defines two preferred directions in spacetime. The first is the direction of $A$ itself. This is the local direction of time where clocks are Lorentz-dilatated. The second is the direction of the Lorentz force on $A$ itself, that is $K^\gamma = \sqrt{\hbar} g_{\alpha\beta} F^{\alpha\gamma} e_A^\beta$, where $e_A$ is the unit vector in the direction of $A$. The direction of $K$ is spacelike and orthogonal to $A$, where rods are Lorentz-contracted. So, if $e_{A\,\mu} = \frac{A_\mu}{\sqrt{g^{\lambda\kappa} A_\lambda A_\kappa}}$ is the unit vector in the directon of $A$ and $e_K$ equally is the unit vector in the direction of $K$, furtheremore $e_1$ and $e_2$ are two unit vectors orthogonal to $A$, $K$ and among themselves, the spacetime metric is $g = \frac{1}{\gamma}\, e_A \otimes e_A - \gamma\, e_K \otimes e_K - e_1 \otimes e_1 - e_2 \otimes e_2$, where $\gamma$ is the Lorentz factor.

$\gamma$ could be a function of $|A|$, $|K|$, $\lambda$ and their gradients. The involvement of $\lambda$, however, is very questionable, since is the multiplier that makes the Lagrangian null. If it was involved, a consistency relation would have to be satisfied to retain the proper functioning. Now enters dimensional analysis, which of course only gives an indication as usual. At this stage, one could argue that one simply has to measure any quantity in its Planckian unit, but it is not very plausible. All the quantities, including the gradient of $\lambda$, are dimensionful. The only pure number is $\lambda$. $|K|$ can be excluded, since it involves $\hbar$. The only quantity that has the inverse dimension of $G$ is $A^2$. But such a factor is needed between the graviational field action and the action of the sources, while there is none in (5) except in $\rho$. So I conjecture $\frac{1}{\gamma} = G\ A^2$, hence

$$g = G\ A \otimes A - \frac{K \otimes K}{G\ A^2 K^2} - e_1 \otimes e_1 - e_2 \otimes e_2\ . \tag{7}$$

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