How fundamental is Newton's Constant?

Gerald Vones

Graz, Austria, European Union
gerald@vones.eu

Abstract

Based on the Born-Infeld field theory it is argued that Newton's constant G appears in a fundamental rotation (boost) symmetry and thus is an element of the Planckian set of units. There is no more fundamental interaction from which G would emerge.

PACS:
Key words:

1 The appearance of Newton's constant

As any physical theory, General Relativity is based on a symmetry which in this case is mediated by Newton's constant G. But this symmetry looks quite strange. In the Hilbert-Einstein action, G converts the Einstein tensor with dimension $\frac{1}{length^2}$ to the stress-energy tensor with dimension $\frac{energy}{length^3}$. Here and in the following I use units where the velocity of light is unity, while I retain G and Planck's constant $\hbar$ as dimensionful. In fact, General Relativity is asymetric in spacetime and energymomentum to the maximum extent possible. It regards the density of the latter as the source of a property of the first. This asymmetry is the deep cause for the incompatibility of General Relativity with quantum physics, and fuels doubt about the foundations of the current view on gravitation. Consequently, there has been a variety of considerations about the nature of G, as Sakharov's metric elasticity of space [1] or the entropic character of the gravitational force [2]. Grosso modo one can say that none of these pictures has substantially illuminated the role of G beyond the state of the art at the early 20th century.

How can G enter the theoretical framwork in a really convincing way? A hint could come from the other two members of the Planckian set, these are the velocity of light and Planck's constant. The velocity of light appears inside the line element of spacetime. If the indefiniteness of the metric is left to another story, there truly is no more simple - or call it fundamental - thing than a rotation or boost symmetry. So the case of the velocity of light is ultimately settled, at least as long as spacetime is intrinsically flat. At the same time, $\hbar$ fits perfectly to the symplectic structure of phase space as it had been known from classical Hamiltonian mechanics. Not only provides it a unit for the phase space volume, rather quantum physics teaches that the phase space volume is countable in this unit. So, the case of $\hbar$ appears as settled in a satisfactory manner as well. G eventually has dimension $\frac{length}{energy}$, so it could enter a line-element after the example of the velocity of light, here however with the resulting space spanned by spacetime and energymomentum. Now, if such a metric symmetry shall exist, in its context energymomentum must must mean a different thing than what is familiar as conjugate variables to positions. There is a strong candidate: (bosonic) fields.

A bosonic field ϕ naturally has dimension $\sqrt{\frac{1}{energy \cdot length^3}}$. In the context of field theory, this usually is multiplied by $\sqrt{\hbar^3}$, what eventually assigns to the field the dimension of energy (not a density)! This looks like a mere convention, however something deep could be behind. Well, for the electromagnetic field, the historic convention has been different simply for the reason that in the Maxwell equations $\hbar$ is invisible because of the masslessness. The gravitational field usually even is defined as a pure number. But any such convention can be changed by multiplying by the appropriate power of the respective Planckian unit.

2 The Born-Infeld Theory

In 1934, Born and Infeld [3] proposed for the action for the electromagnetic field $L = \frac{1}{G^2\hbar}\left[\sqrt{-\det(g + G\sqrt{\hbar}\, F)} - \sqrt{-\det g}\right]$, where g is the metric of spacetime, and F is the electromagnetic field tensor which got assigned the historic dimension $\sqrt{\frac{energy}{length^3}}$. It must be clarified that in [3] obviously units with $c = G = \hbar = 1$ were used, so these constants do not appear there (see in particular their equation (2.4.)). Since the topic here is the role of G, it is central to have G and $\hbar$ written out explicitely, so I supplemented them. By expanding the determinant, one can see that this action includes the coupling of the electromagnetic field to the metric.

At the technical level things are a bit subtle, in particular what regards the pure helicity one of the electromagnetic field. But here the focus shall be on the quintessence, that is the way G enters the framework. For this, it is sufficient to regard the simplest case of a massless real scalar ϕ. Then a Born-Infeld type action emerges from the line element of a 5-dimensional embedding space

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu \pm G^2\hbar^3 \mathrm{d}\phi^2 \ , \tag{1}$$

where the indices are raised and lowered by means of g. This is to be supplemented by the postulate that the 4-dimensional submanifold produced by the field equation $\phi(x^\mu)$ - furtheron called the "field manifold" - shall be stationary. For any number of fieldlike degrees of freedom, the field manifold is 4-dimensional, since the number of field equations rises correspondingly. On such a field manifold, there exists the metric induced from embedding $\gamma_{\mu\nu} = \frac{\partial X^a}{\partial x^\mu}\frac{\partial X_a}{\partial x^\nu}$. X^a are the coordinates in the embedding space. The latin index is to be raised and lowered by means of the embedding space metric. The greek indices run from 0 to 3, and the x^μ can be identified with those on spacetime. The volume element of such a field manifold is given by $\sqrt{-\det\gamma}\ \mathrm{d}^4x$. The Lagriangian is the volume multiplied by the vacuum energy density

$$L = \pm\frac{1}{G^2\hbar}\sqrt{-\det\gamma}\ . \tag{2}$$

Again for the case where ϕ is just a real scalar, the sign must be equal to the metric signature of ϕ. In this case the induced metric is $\gamma_{\mu\nu} = g_{\mu\nu} \pm G^2\hbar^3\partial_\mu\phi\partial_\nu\phi$. This yields $L = \pm\frac{1}{G^2\hbar}\sqrt{1 \pm G^2\hbar^3\partial_\mu\phi\partial^\mu\phi} \approx \pm\frac{1}{G^2\hbar} + \frac{1}{2}\hbar^2\partial_\mu\phi\partial^\mu\phi$, where the indices are raised and lowered by means of g.

3 Vacuum energy and Gravitation

With the above, the aim of this note is already achieved. If the Born-Infeld ansatz is correct, then G does appear in a fundamental symmetry and consequently is an element of the Planckian set. Like c and $\hbar$ it is just a representation of the pure number 1, emerging only because - unnecessary - dimensionful concepts had been introduced. There is no more fundamental mechanism from which G could emerge.

Nevertheless, I would like to add some observations since there is a possibility to gain a new sight on gravitation. The formulation in terms of equations (1) and (2) removes the not understandable linearity of nongravitational field theory (I do not speak about Yang-Mills theories, where the nonlinearity originates from the internal symmetries). Furthermore, the Planckian energy density $\frac{1}{G^2\hbar}$, that is one Planck mass per Planck length cubed, is the only plausible candidate for the absolute value of the vacuum energy density. It is huge enough to explain why the nonlinearities have not been observed so far. Of course, plugging it into the equations of General Relativity supplemented by the Cosmological Term would yield a nonsense result. But the pragmatic subtraction made by Born and Infeld now finds a very plausible explanation: Like the rest mass of an electron cannot be converted to radiation because of the additional conservation law for the lepton number, the energy density of the nongravitational fields cannot be converted to gravitational energy density because of an additional conservation law. The exception in Special Relativity is an annihilation event between electron and positron, and an analogue in the gravitational context obviously took place at the big bang (where, however, the energy of the "anti-universe" actually counts negative).

This leads over to the field manifold of the gravitational field itself. It must live in embedding space as well, while at the same time being substantially different from any nongravitational field manifold. Again, after the example of Special Relativity, there is a very clear conclusion: The field manifold of gravitation is null, i.e. $\det \gamma_{gr} = 0$. In 1998, Deser and Gibbons [4] proposed an action for the gravitational field straight after the method of Born-Infeld. This reads $L = \sqrt{-\det(ag + bR + ...)}$, where a and b are some (partially dimensionful) constants and R is the Ricci tensor. The dots indicate that terms quadratic or higher in curvature might follow. From the above I conclude that central task of these additional terms, in what ever order in curvature, is to make the determinant vanishes.

References

[1] Sakharov AD, Sov.Phys.Dokl. 12, 1040 (1968)

[2] Verlinde EP, JHEP 1104:029 (2011) (arXiv:hep-th/1001.0785)

[3] Born M, Infeld L, Proc.R.Soc.Lond. A 144, 425 (1934)

[4] Deser S, Gibbons GW, Class.Quant.Grav. 15, L35 (1998) (arXiv:hep-th/9803049)