Why Are There Laws of Nature?

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Abstract

It is argued that solving the enigma of information theory via a self-consistency requirement implies straightforwardly that the information content of nature is logarithmically small, which means that there are laws of nature.

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Invariances, alias symmetries, alias redundancies, alias laws of nature, are the basic subject of the physical sciences. They allow us to know parts of nature there and then – in whatever space may be relevant – from what they look like here and now. For example, if we measure the energy of a closed classical system at one instant, this means that we know this quantity forever.

There are numerous interesting aspects of such invariances, with Noether's theorem as a key. However, the most pressing questions reach down even deeper, to where the only concept left is that of information itself. Why is nature not completely random? To what degree is nature redundant?

We define "nature" here to be everything relevant to the following discussion, be it the universe or more. To the extent to which the concept of dimensionality is appropriate at all, nature shall span all dimensions including the time-like degrees of freedom. So, when I address the "state" of nature, this is nothing dynamic. To illustrate the situation, I shall speak about a "screen" and call the element of information a "pixel". This shall serve as a methaphor only, with no direct relation to holography. There is evidence that the element of information is a two-valued "bit", but a base of 2 for the exponential function is not crucial for the following conclusions. The symbols "exp" and "log" can be read in an abstract sense.

So, one has a screen showing an image with symmetries, which means an image with less information than the maximum possible: the image can be produced by a computer code consuming fewer pixels than the screen has available. This leads to the basis of information theory; indeed, to the very meaning of the idea of "information". Though omnipresent in modern physics (in the entropy of black holes, information loss [1], entanglement [2] and many more areas), information theory is not yet free from a fundamental enigma. Intuitively, one would say that the information associated with a screen is what one sees there, for example a sketch of the expanding universe. However, such semantic content should correctly be considered as the factual state of the screen, while the information content is defined to be the number of pixels – which is the logarithm of the number of possible states. Now, if these two aspects of the information coincide, this conflict is solved in a self-consistent manner. This means that the factual state of the screen displays its number of pixels. Translated back, *the one and only state of nature encodes its number of degrees of freedom.*

Something of that kind can occur in everyday life when a screen is used to advertise itself. Suppose the screen displays a message such as "5 Megapixel Screen – Promotion". Addressed to humans, this message contains some linguistic encoding and an additional message. The fonts are human-sized as well, and consume a lot of pixels. But, in a strict, "puritanical" sense, the number of pixels needed to encode a number is only the logarithm of that number. The rest of the screen is filled with redundant content. This yields the numbers characterizing the screen shown in Table 1.

There is a clear connection to standard thermodynamics and quantum theory, where the residence of information is phase space. Although these theories do not regard time t and energy E as a pair of conjugate variables in the strict sense, this is the most important ensemble from the physical point of view. Basically, a massive object just is doing nothing but acting over its lifetime t_{object} via its rest energy E_{object}. And it is well verified that there is a related uncertainty relation: The shorter an object lives, the more uncertain is its energy. So the screen is a rectangle bounded by the lines

Table 1: Numbers characterizing the screen described in the text.

Number of pixels	N
Number of possible states	$\exp N$
Number of factual states	1
Semantic content of factual state	"N"
Nonredundant information	$\log N$

$E = 0$, $E = E_{object}$, $t = 0$, $t = t_{object}$. The rectangle is homogeneoulsy filled with pixels at macroscopic scales, what means that the number of pixels N is the area S, divided by a unit of action h.

The number of possible configurations of the pixels is $\exp \frac{nS}{h}$, where n is the number of different states a pixel can be in. This indeed is what quantum theory assignes as the weight factor in the path integral apart from the strange fact that n is the imaginary unit indicating that the mechanism is quite unreal. For clarity it can be added that the path integral extends over the ensemble of screens with any upper border, not only the classical horizontal line $E = E_{object}$. However this is not the space of possibilities addressed here, which rather deals with the state of the pixels in case of one definite upper border.

Applying the above conclusions, there is only one single state of the screen. And this state encodes the number of pixels, what consumes only $\log \frac{S}{h}$ pixels if n is set real unity. Statistical thermodynamics teaches that not the phase space volume (the area of the screen) is to be identified with the entropy, rather the logarithm of that quantity is. In other words, *entropy is non-redundant information, and this is logarithmically small.* [3] presents a drawing showing God pointing at a tiny portion of phase space. This shall illustrate exactly how incredibly small the entropy of nature is (to be precise, the drawing refers to the entropy of our universe at early times – I shall not discuss that topic, apart from repeating that I see information as nondynamic).

What can be said about N_{nature}, the number of degrees of freedom of nature as a whole? Table 2 gives an overview.

The primary guess could be that the phase space volume is infinite though countable, so it has the mightiness of the natural numbers $\beth_0 \equiv \aleph_0$. This, however, is impossible since the logarithm of $\beth_0$ does not exist. There are two alternatives, none of which should be excluded before the meaning of quantization is ultimately clarified - in particular whether the phase

Table 2: Variants for the values of N_{nature}.

#	N_{nature}	$\log N_{nature}$
1 (impossible)	$\beth_0$	not existent
2	$\beth_1$	$\beth_0$
3	finite	finite

space volume is indeed countable. The first alternative is $N_{nature} = \beth_1$, the mightiness of the continuum, while its logarithm has mightiness of the natural numbers. This suggests some subtle interplay between the classical continous perception of nature and entropy which is countable. The second alternative is that both N_{nature} as well as its logarithm are finite.

References

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