Why Are There Laws of Nature?

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Abstract

It is argued that solving the enigma of information theory via a self-consistency requirement implies straightforwardly that the information content of nature is logarithmically small, what is the origin for the existing laws of nature.

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Invariances, alias symmetries, alias redundancies, alias laws of nature, are the basic subject of the physical sciences. They allow us to know parts of nature there and then – in whatever space may be relevant – from what they look like here and now. For example, if we measure the energy of a closed classical system at one instant, this means that we know this quantity forever.

There are numerous interesting aspects of such invariances, with Noether's theorem as a key. However, the most pressing questions reach down even deeper, to where the only concept left is that of information itself. Why is nature not completely random? To what degree is nature redundant?

By definition, "nature" shall be everything relevant to the following discussion, be it the universe or more. To the extent to which the concept of dimensionality is appropriate at all, nature shall span all dimensions including the time-like degree(s) of freedom. Symmetries involving time obviously exist and are most important since they allow to predict the future. So, when I address the "state" of nature, this is nothing dynamic. To illustrate the situation, I shall speak about a "screen" and call the element of information a "pixel". This shall serve as a metaphor only with no intended relation to holography, while some elements of holography will pop up. Furthermore, the base for the exponential function is not crucial for the core of the following conclusions. The symbols "exp" and "log" can be read in an abstract sense.

So, one has a screen showing an image with symmetries, which means an image with less information than the maximum possible: The image can be produced by a computer code consuming fewer pixels than the screen has available. This leads to the basis of information theory; indeed, to the very meaning of the idea of "information". Though omnipresent in modern physics (in the entropy of black holes, information loss [1], entanglement [2] and many more areas), information theory is not yet free from a fundamental enigma. Intuitively, one would say that the information associated with a screen is what one sees there, for example a sketch of the expanding universe. However, such semantic content should correctly be considered as the factual state of the screen, while the information as such is a pure number, the "amount of" information. This simply is the number of pixels – which is the logarithm of the number of possible states. Now, if these two aspects of the information coincide, this conflict is solved in a self-consistent manner. This means that the factual state of the screen displays its number of pixels. Translated back, *the one and only state of nature encodes its number of degrees of freedom.*

Something of that kind can occur in everyday life when a screen is used to advertise itself. Suppose the screen displays a message such as "5 Megapixel Screen – Promotion". Addressed to humans, this message contains some linguistic encoding and an additional message. The fonts are human-sized as well, and consume a lot of pixels. On the other hand, 5 million is a quite special number with low information content as is obvious from its decadic representation. But in the genuine case, the number of pixels needed to encode a number is (the following integer to) the logarithm of that number. The rest of the screen is filled with redundant content. This yields the numbers characterizing the screen shown in Table 1.

It can be added that any semantic content is commonly regarded as requiring an observer interpreting the information, figuratively speaking a person watching the screen. But since the semantic content is just the

Table 1: Numbers characterizing the screen described in the text

Number of pixels	N
Number of possible states	$\exp N$
Number of factual states	1
Semantic content of factual state	"N"
Nonredundant information	$\log N$

number of pixels, there is little need for any comprehension by an observer. Even more, there are observer-indepenent preferred representations: Either the base of the exponential is 2 (the smallest integer doing the job) in the case of coutability, or it is e in the case of the continuum. In contrast a base of 10 induced by human anatomy certainly has no preferred role.

To address the possible values of N, I first refer to the mightiness. Table 2 lists the possibilities. In any case, it cannot be infinite though countable - having the mightiness of the natural numbers $\beth_0$ -, since the logarithm of $\beth_0$ does not exist. The first alternative is $N_{nature} = \beth_1$, the mightiness of the continuum, while its logarithm is the mightiness of the natural numbers. Higher Beth numbers are against evidence, since quantum physics teaches that countability is a characteristicum of nature at least in some way. Finally, both N_{nature} as well as its logarithm can be finite. In this case, N_{nature} is integer. Its logarithm is integer by construction, since the following integer is taken.

Table 2: Variants for the values of N_{nature}

#	N_{nature}	$\log N_{nature}$	remark
1	$\beth_0$	not existent	impossible
2	$\beth_1$	$\beth_0$	
3	$\beth_n \quad n > 1$	$\beth_{n-1}$	against evidence
4	finite	finite	integer

Variant 2 reminds of the apparent subtle interplay between the classical continuous perception of nature and quantization. Nevertheless, it is poorly plausible. The reason is the scale invariance of the continuum: An arbitrarily tiny portion of the relevant space would contain infinitely much nonredundant information. Quantum physics supplements the symplectic structure of phase space known from the classical theory with a unit of volume. Such elementary cells of phase space can be regarded as bits, where the volume element can be oriented in the one or the other way. Combination with gravitational physics allows to extend such elementary cells beyond phase space. In particular, the horizon area of a black hole can be regarded as made up of elementary volume elements with size of the Planck area. Bekenstein's formula relates this area to the associated entropy. Although quantum physics is not yet understood completely while quantum gravity is pending, it widely suggests to associate the quantized phase space volume as well as the gravitational entropy with N_{nature} rather than with its logarithm. So I conclude that nature is finite.

The self-constency relation implies that the nonredundant information is logarithmically small. [3] presents a drawing showing the Creator pointing at a tiny portion of phase space. This shall illustrate exactly how incredibly small the entropy of nature is. To be precise, the drawing refers to the entropy of our universe at early times, while under the current view entropy is a - monotonically increasing - function of time. This is the next enigma: If information is not dynamic, how can the impression of dynamics emerge at all? Well, redundancies can be encoded in numerous ways. It is a remarkably self-contradictory though succesful exercise to write a computer code generating long-period sequences of numbers looking "random" to a high extent. As one of the simplest examples, a linear congruence generator based on the mapping $x_n \rightarrow x_{n+1} = (ax_n + b)\ mod\ m$ can have a period with lenght of the modulus m if the constants a and b are chosen appropriately. So, the period is exponentially larger than the number of bits necessary for implementing the generator. The code is manifestly of low information, while any output string is of equal low information, though not manifestly. It is quite plausible that at the header of the file discribing nature stands the code, while later down the file the code becomes more and more diluted by its own output. This sheds a completely new light on the various information paradoxes. There would neither be a trapping of information in black holes or elsewhere nor would there be a creation of information due to quantum processes. Only the loss of the manifest visibility of the actually tiny information is progressing with time.

Our universe is regarded to either evolve towards a black hole [3] or being one [4]. In both cases, the lower bound for the associated entropy is the mass squared as observable today. References give slightly different values, but this is quite irrelevant and one can use $N > 10^{123} = 2^{409}$. There exist dynamic variables like positions or fields together with their associated momenta. Over time, they invade the associated phase space and produce the said entropy. However as argued above, dynamics only removes the manifest visibility of the low entropy. Not the area of the screen is to be identified with the nonredudant information, only its logarithm is. Using the said lower bound would result in only a few hundread bits carrying all the information about nature. However, one must bear in mind that this lower bound is rather generous. For example, if the picture given in [4] holds, the essential scales grow proportional to time over an unknown further duration. Second, it is far from proven that our universe is nature in its entirety.

References

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