Why Are There Laws of Nature?

Gerald Vones

Graz, Austria
gerald@vones.eu

Abstract

It is argued that resolving the enigma of information via a self-consistency requirement implies straightforwardly that the information content of nature is logarithmically small, what is the origin for the observed laws of nature. Information paradoxa are avoided in this way.

PACS: 01.55.+b
Keywords: Information, Symmetries, Laws of Nature

1 Introduction

Redundancies, alias symmetries, alias invariances, alias laws of nature, are the very subject of the physical sciences. They allow to know parts of nature there and then from what they look like here and now. There are numerous interesting aspects of such invariances, with Noether's theorem as a key. Well, things may be more subtle where no dimensionful entity such as spacetime yet has emerged. The quantum aspects of nature complicate the situation further, while they make quite clear that "information" is the central entity. In fact, the most basic questions about nature go down to the level where the only concept left is that of information itself: Why is nature not completely random? To what degree is nature redundant and what is the very reason for such redundancies?

With information so much at the center, in the following section I shall start the discussion about the laws of nature with a wrap up on the current knowledge on those concept. In the next sections I shall then apply these conclusions first to a toy case, consecutively to nature as such.

2 The Enigma of Information

A textbook on information theory starts with a reference to the Iron Age [1]: A blacksmith of those period may have been very skillful in handling iron, while having no idea of what iron actually is. One needs insight down to the level of atoms to comprehend the nature of the different substances. Living in the Information Age, the book argues, we might be equally ignorant of what information is. Then the book continues with concepts in whose core is the semantic content of some elements of information.

So the question is put: Do we know what information is? To find an answer, as an illustration, I shall use a television screen made up of N pixels. Furtheron, I shall call the element of entropy a pixel, occasionally bit or digit. In fact, the base of the exponential (logarithmic) function is not important for the conclusions. The symbols "exp" and "log" can be read in an abstract sense. I shall use the abbreviation TV for the apparatus under consideration to avoid the notion of a screen, which suggests some specific meaning in current theoretical physics. It is important to clarify that the TV is regarded as extending over the two spatial dimensions only, not over time. So the TV can show an image rather than a movie.

One can distinct three levels of information theory:

- Firstly, information theory just is theory of entropy, and no distinction between the two concepts exists. This can be seen from the complete identity of the basic equations with such familiar from statistical physics. The Shannon entropy reads $S = -\sum_{w=1}^{W} p_w \log p_w$, where p_w is the probability for the system being in state w out of W different states. In the simplest case where all the p_w are equal, the Boltzmann entropy $S = \log W$ arises. In this sense, information theory is a well funded, transparent and valuable tool for handling bits.

- Secondly, information theory is dealing with the interplay of possibility and factuality. Due to this understanding, there is a factual state w_{actual} with the probabilities collapsing - and be it only due to the reckognition by a formerly ignorant observer - towards the Kronecker delta $p_w \rightarrow \delta_{w\, w_{actual}}$. With this collapsed probability function inserted into the Shannon formula, no entropy is left. So one says that the entropy has been sucked out as information and usually denotes it as $I = S(\delta_{w\, w_{actual}}) - S(p_w) = -S(p_w)$, where the overall sign is a matter of taste. In our example, the TV can take on $W = \exp N$ different states with equal probability, so the entropy just is $S = N$. However, the TV does not take on the superposition of all these possible states, rather it takes on one specific factual state. In this sense, information is a rather strange concept related to a number of paradoxa, be it in the classical case, be it in the quantum case.

- Thirdly, information theory deals with the so called semantic content. A well functioning TV does not take on any factual state, rather it takes on a state showing something "sensible". An observer shall not only undergo an experience, rather this shall be an "aha" experience. Semantic content is what reference [1] tries to formalize with methods reminding of propositional calculus. Importantly however, such content is completely separated from the concepts of information discussed above. The TV showing an artpiece of Picasso or showing some type-written tax declaration only have in common that the TV is of equal size, that means of equal N, thus equal I - but this is the last criterion one would regard as essential with relation to semanticity. In this sense, information theory is decoupled from the quantitative description by means of natural sciences and mathematics.

From this I conclude that the answer to the introductory question clearly is a *no*. We do not know what information is, despite of its omnipresence in modern physics (in the entropy of black holes, information loss [2], entanglement [3] and many more areas). If the enigma of information is to be solved, firstly the conflict between possibility and factuality is to be removed, secondly the semantic content must be established as a mathematical object - a number - related to the entropy in a different way than I is.

3 A Self - Advertiser

Imagine the following situation which can occur in everyday life. Entering a store for electronic equipment, you see a TV displaying a message such as "5 Megapixel TV – Promotion". Addressed to humans, this contains some linguistic encoding and an additional semantic element. The fonts are human-sized as well, and consume a lot of pixels. On the other hand, 5 million is a quite special number with low information content as is obvious from its decadic representation. But in the genuine case, the number of pixels needed to encode a number is the logarithm of that number. The rest of the TV can be filled with redundant content. This yields the numbers characterizing the TV shown in Table 1, where I supplemented an additional column referring to the corresponding thermodynamic quantities.

Table 1: Numbers characterizing the TV described in the text, and corresponding thermodynamic quantities

quantity	value	thermodynamic correspondance
Number of pixels	N	Entropy
Number of possible states	$\exp N$	Phase Space Volume occupied
Number of factual states	1	
Semantic content of factual state	"N"	
Non-fallacious information	$\log N$	

On this basis, it is very easy to comprehend that *the enigma of information is resolved if and only if the semantic content of a message is the related number of degrees of freedom. For short, information is the logarithm of entropy.* This is a self-consistency relation with the result that the non-fallacious information is logarithmically small compared to the degrees of freedom available. All the rest is redundant, that means it is fallacious information.

In quantitative terms, a little further sharpening appears as feasible. What is the most straightforward implementation of the redundancy? This is to repeat the message again and again. But then the message need not to encode the number of bits, it is sufficient if it encodes the number of repetitions. This is in fact the way to completely separate the information-carrying elements from the redundancies: All the replicas are exactly the same, while any such - incompressible - replica contains the entire information. So, if the number of replicas is denoted as R and the information is denoted as I, we end up with $I = \log R$ and

$$N = S = R \cdot \log R \;=\; I \cdot \exp I \; . \tag{1}$$

It shall be added that when it comes to the details, there are subtleties, which however do not disturb the big picture. For the TV everything is in terms of natural numbers, with the number of pixels needed to encode a number factually being a step function. For the number 1, already one bit is needed and the binary code is 1 - despite of the fact the the logarithm of 1 is 0. 2 needs two bits with code 10, while 3 also needs 2 bits with code 11. And so on, this is just the familiar binary representation.

4 Subsystems

Since information now is the logarithm of entropy, the case of subsystems must be reconsidered. The first step is to adapt the addition law for entropy to be in line with equation 1. R_k shall denote the number of replicas associated with subTV number k with $R := \sum R_k$, where the sum runs over the entire ensemble. Furthermore $p_k := \frac{R_k}{R}$, which quasi is the probability that any given replica is associated with subTV k. As a mathematical identity, $R \log R = \sum R_k \log R_k - R \sum p_k \log p_k$, where the latter term is the Shannon entropy of a total number of R elements. So the relation assumes the plausible form $S = \sum S_k + S_{Shannon}$. Consequently for such an ensemble, the number of replicas is additive while the number of pixels aquires the correcting Shannon term.

The more subtle problem is the behaviour of I, since this now is not the entropy or the number of replicas, rather the logarithm thereof. If all the subsystems stood alone for theirselves, their information contents would add up to $\sum \log R_k$, consequently the exponential of the summed up information would be $\exp \sum I_k = \prod R_k$. Rather, since actually R is additive, the exponent of the corresponding total information is $\exp I = \sum R_k$. The product always is greater than (or, in one exceptional case equal to) the sum except when the overwhelming majority of subTVs only represents 1 bit. But such case first is quite irregular as discussed before, second it is one of obviously low information content. From this it follows that the R_k cannot be independent quantities. There must be an algorithm of length I which determines them. Though this is quite a trivial insight, it makes clear what the division to subTVs actually must mean, namely that an independent appearance of the R_k is fallacious.

5 Nature and the Quantum

To use the example of the TV to learn about nature as a whole, the following definition shall be used even if it close to a logic circle: Nature is the entirety of what is relevant for the considerations here. It is the entity whose number of degrees of freedom is encoded as discussed. As an important difference to the example of the TV, time now is among the basic degrees of freedom determining the number of pixels of nature, N_{nature}. Nature spans all dimensions if the concept of dimensionality is relevant at all. There may be some parts of nature where it makes no sense to speak about dimensions, there may be some parts where more than one timelike dimension exist. For the part familiar to us, this means that nature spans the three spacelike dimensions plus time. So the state of nature is nothing dynamic, rather it is the entirety of what has ever been, is, and will be. As an example for a subsystem, a black hole comprises the entire history of the stuff that over some duration of external time is inside the hole, while first it had fallen in and later will be irradiated off.

So, as a tautology, nature is in one and only one factual state. The inclusion of time also removes the difficulties caused by quantum aspects. Either there is a wave function whose evolution - which is unitary, as far as we know - comprises the entirety of nature. This wave function can be described in terms of classical bits (C-bits), which hold the respective set of quantum numbers. Or there actually is something that must be added to the wave function to make it collaps. But such a processes just are further degrees of freedom which again can be encoded in terms of C-bits: These bits hold the outcome of the measurements, which always are C-numbers. In fact, there have already been found resons to assume that underneath the quantum appearance, there actually is a mechanism like cellular automata [4].

6 Information and Illusion

Obviously, nature has symmetries. Symmetries in time are among those, and they even are the most important of all since they allow to predict the future. But so long it has been miracolous where such redundancies originate from and it has been unknown what this means in quantitative terms. Now a highly plausible answer can be read off from table 1 with N replaced by N_{nature}.

But, how can the information content of nature be that small? Despite of the reckognizable symmetries, the major part of nature does not look as governed by laws, with the black hole as the most impressive example. The stuff such a hole is constituated of looks completely arbitrary, consequently the entire Bekenstein-Hawking entropy seems to be necessary to encode it [5]. This leads to the black hole information paradox, since the hole only exhibits its macroscopic parameters. The discussion in the last section allows a radical insight removing the paradox: In fact, only the macroscopic parameters of the hole are non-fallacious information, determining everything else. The same is the case with a system underlying classical statistical physics. Experience teaches that the macroscopic parameters are sufficient to know the relevant aspects of the system. The insight gained here is that these parameters not only encode the relevant aspects, they encode all of the aspects. The surprised one may be that these systems contain that little information, actually they contain even less - they correspond to subsystems as discussed, so their macroscopic parameters are mutually correlated.

This leads over to our universe as another example for a subsystem of nature. It is questionable whether its estimated parameters are relevant for the sake of calculating the information. In particular, the contribution of dark components is unknown. Even more grave, in the light of recent developments one might conjecture that that the dominant contributions come from the cosmic horizon. Well, if the mass visible inside the horizon is inserted, references give slightly different values, what however is of minor relevance and one can use $N_{universe} \approx 10^{123} = 2^{409}$. This would result in only a few hundread bits encoding all the non-fallacious information. In any case, that the non-fallacious information of the stuff inside the universe is very small can be seen from the highly ordered initial state. [6] presents a drawing showing the Creator pointing at a tiny portion of phase space. This shall illustrate exactly how incredibly small the initial phase space volume was compared to the phase space volume available. This suggests that at the big bang the non-fallacious information was directly visible, while over time fallacious contributions are coming along. This fallacious information must be illusive, that means the redundancies appear in a camoufled manner. Abstracted from the yet mentioned idea of cellular automata, I would like to continue the discussion without excluding any possibilty.

In fact, there are numerous ways of filling up the TV. One possibility is to repeat the number of pixels again and again. Another possibility is to make the background monocolorous - and so on. While these examples evidently exhibit the redundancy, there are much more tricky alternatives. It is a remarkably self-contradictory though succesful exercise to write a computer code generating long-period sequences of numbers looking "random" to a high extent. The simplest example is a multiplicative linear congruential generator based on the mapping $x_n \rightarrow x_{n+1} = (ax_n + b)\ mod\ m$, with the constants chosen carefully. So called Fibonacci generators are another frequent choice for practical use [7]. The essential parts of such a machine used in practical terms need some hundread bits for encoding, and they can produce a sequence of pseudo-random numbers which is the exponent of some hundread bits long. So the pseudo-random sequences are exponentially longer than the code necessary.

If the bitcode of the generator itself is interpreted as a number N (whose encoding needs $\log N$ pixels), then this machine produces order of magnitude N pseudo-random numbers. One can use this sequence to fill the entire TV with a seemingly random pattern of pixels. The corresponding information of magnitude N is fallacious. Something of that kind must occur in our universe, where at the header stands the code, which further down becomes more and more diluted by its own output. The evident redundancies remaining are the conservations of the Noether currents.

7 Cardinality

For the TV everything is countable, even finite. To discuss this aspect for nature, the corresponding possibilities are listed in Table 2, where the number of degrees of freedom of nature is notated as N_{nature}. $\beth_0$ is the cardinality of the natural numbers, while $\beth_1$ is the cardinality of the continuum. Neither of the first two variants can be realized, since the logarithm of $\beth_0$ does not exist. The first alternative is $N_{nature} = \beth_1$, while its logarithm is the cardinality of the natural numbers. Higher Beth numbers are against evidence, since quantum physics teaches that countability is a characteristicum of nature at least in some way. Finally, both N_{nature} as well as its logarithm can be finite. In this case, N_{nature} is integer. Its logarithm is integer by construction, since the following integer is taken.

Table 2: Variants for the values of N_{nature}

#	$\exp N_{nature}$	N_{nature}	$\log N_{nature}$	remark
1	$\beth_0$	not existent	not existent	impossible
2	$\beth_1$	$\beth_0$	not existent	impossible
3	$\beth_2$	$\beth_1$	$\beth_0$	eventually excluded
4	$\beth_w \quad n > 2$	$\beth_{n-1}$	$\beth_{n-2}$	against evidence
5	finite	finite	finite	integer

Variant 3 reminds of the apparent subtle interplay between the classical continuous perception of nature and quantization. Nevertheless, it is poorly plausible. The reason is the scale invariance of the continuum: An arbitrarily tiny portion of the relevant space would contain infinitely much nonredundant information. Planck's constant supplements the symplectic structure of phase space known from the classical theory with a unit of volume. Although quantum physics is not yet fully comprehended, there is much evidence that the phase space volume is countable. So the cardinality of $\exp N_{nature}$ is at most $\beth_0$, and from the first row of table 2 now follows that no infinity can occur at all. Furthermore, in current gravitational physics elementary cells can be found for entropy, which exist in ordinary space. In particular, the 2-dimensional horizon of a black hole can be regarded as made up of elementary pieces with size of the Planck area. Bekenstein's formula [8] relates this area to the associated entropy, with the elementary pieces of the horizon acting as "bits". This means countability for N_{nature} in the case of gravitation, and now the second row of table 2 implies that no infinity can occur. So variant 5 is the only survivor and I conclude that N_{nature} *is finite.* This makes the analogy with the TV complete.

8 Conclusion

The question of how the laws of nature look, was not touched in this discussion, which rather was purely in terms of the amount of redundancy. Despite of this and any further yet uncomprehended aspect, the following conclusions can be drawn:

- The laws of nature originate from a self-consistency requirement for information;
- The non-fallacious information is logarithmically small, everything else is governed by laws producing redundancy;
- Apart from a few evident excemptions (Noether currents), the redundancies are camoufled;
- In terms of the non-fallacious information, there are no information paradoxa;
- Nature is finite.

References

[1] Devlin KJ, *Logic and Information* (Cambridge University Press, 1991)

[2] 't Hooft G, arXiv:0604008 [quant-ph] (2006)

[3] Bub J, Quantum entanglement and information, *The Stanford Encyclopedia of Philosophy* (Zalta EN ed., Winter 2010 edition)

[4] 't Hooft G, arXiv:1405.1548 [quant-ph] (2014)

[5] Hawking SW, Nature 248, 5443 (1974)

[6] Penrose R, *The Emperor's New Mind* (Oxford University Press, 1989)

[7] Lang CB, Pucker N, *Mathematische Methoden in der Physik* (Springer Spektrum, 2005)

[8] Bekenstein JD, Phys. Rev. D 7, 2333 (1973)