Linearity

Gerald Vones

Graz, Austria, European Union, gerald@vones.eu

Abstract

It is argued that the quantum world of nongravitational fields lives on a null manifold, what explains its linearity.

That nongravitational fields propagate linearly in spacetime is familiar and appreciated. However, from a full geometric unification of all physical concepts, wouldn't field equations have to describe extremal surfaces? Special Relativity suggests that null surfaces are the key. There the action increment is $\mathrm{d}S = -m\mathrm{d}\tau = -m\sqrt{1-v^2}\mathrm{d}t$ with standard meaning of the symbols. This can be rearranged as $\mathrm{d}S = -\frac{m}{\sqrt{1-v^2}}(1-v^2)\mathrm{d}t$. For singular (lightlike) particles, both numerator and denominator of the first factor vanish, however the ratio as such stays finite, namely it is the frequency times Planck's constant $h\nu = \frac{m\mathrm{d}t}{\mathrm{d}\tau}$.

Indeed, the action in field theory is null, since the underlying dispersion relation reads $p_\mu p^\mu - m^2 = 0$. The mass m shall be finite, while massless particles are left aside here. Furthermore, only the flat space kinetic term is regarded, so the indices are raised and lowered by virtue of the Minikowski metric where the signature shall be (+ - - -). The dismissed aspects need special treatment, but are not the point here. One can insert $p^\mu = m\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$ and $m = -\frac{\mathrm{d}S}{\mathrm{d}\tau}$. This yields the null line element $0 = \mathrm{d}\sigma^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu - \frac{\mathrm{d}S^2}{m^2}$, where I divided by m^2 to make the metric of spacetime unity. So under a new alternative point of view one could cease to regard spacetime as the arena, rather one could say that the object lives on a 4-dimensional null manifold embedded in a 5-dimensional flat space. The spacetime degrees of freedom can be continued to be used as the independent coordinates. From the embedding equation $S(x^\mu)$ one can build the total differential $\mathrm{d}S = \frac{\partial S}{\partial x^\mu}\mathrm{d}x^\mu$. Together with the vanishing of the above line element, this yields the identity $\frac{\partial S}{\partial x^\mu} = m^2\frac{\mathrm{d}x_\mu}{\mathrm{d}S} + a_\mu$. The relation can be seen as describing an associated bundle of null trajectories. a is orthogonal to the path as projected on spacetime $a_\mu \mathrm{d}x^\mu = 0$, else arbitrary and shall be set zero here.

For the above to make sense, however, the mass m must have a fundamental meaning. This is the case if unification of spacetime with some massive field ϕ is sought. Since it is known that S is the phase of the field, the line element becomes

$$\mathrm{d}\sigma^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu - \frac{\mathrm{d}\ln\phi\,\mathrm{d}\ln\phi^*}{m_\phi^2} = \mathrm{d}\tau^2 - \frac{\mathrm{d}r^2 + \mathrm{d}S^2}{m_\phi^2}\ , \tag{1}$$

where m_ϕ is the mass of the field quantum. I already equipped the field with a nontrivial absolute value in addition to the phase to arrive at a complex scalar $\phi = \exp(r + \imath S)$, what is straightforward. In this case, in terms of real degrees of freedom counted separately, the flat embedding space (x^μ, r, S) is 6-dimensional. The 2 embedding equations $r(x^\mu), S(x^\mu)$ solving the field equations again define a 4-dimensional null submanifold. The null trajectories remain 1-dimensional and the relations emerging from $\mathrm{d}\sigma^2 = 0$ now read $\frac{\partial \ln\phi}{\partial x^\mu} = m_\phi^2 \frac{\mathrm{d}x_\mu}{\mathrm{d}\ln\phi^*}$ and complex conjugate, or in components $\frac{\partial S}{\partial x^\mu} = m_\phi^2 \frac{\mathrm{d}x_\mu \mathrm{d}S}{\mathrm{d}r^2 + \mathrm{d}S^2}$ and $\frac{\partial r}{\partial x^\mu} = m_\phi^2 \frac{\mathrm{d}x_\mu \mathrm{d}r}{\mathrm{d}r^2+\mathrm{d}S^2}$.

The induced metric on the embedded null manifold is hermitean $g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{m_\phi^2}\partial_\mu \ln\phi \partial_\nu \ln\phi^*$ with real determinant $\det g = -1 + \frac{\partial_\mu \ln\phi \partial^\mu \ln\phi^*}{m_\phi^2}$. Now, after the example of Special Relativity, the field action increment is written as $\mathrm{d}A = \frac{\rho}{\sqrt{|\det g|}} \det g\ \mathrm{d}^4x$, where ρ is the classical mass density (particle mass per volume of 3-space). Again, in the singular case both ρ and $\sqrt{|\det g|}$ vanish, while their ratio is finite. The value is well known, namely it is the probabilistic density $m_\phi^2 \phi^* \phi$ what makes $\mathrm{d}A$ the correct action increment. The similarity to Special Relativity is even closer if the expression is written in terms of the trajectories. In this case $|\det g| = 1 - m_\phi^2 \frac{\mathrm{d}x_\mu}{\mathrm{d}\ln\phi} \frac{\mathrm{d}x^\mu}{\mathrm{d}\ln\phi^*} = (\frac{\mathrm{d}\sigma}{\mathrm{d}\tau})^2$, consequently $\frac{\rho}{\sqrt{|\det g|}} = \frac{\rho \mathrm{d}\tau}{\mathrm{d}\sigma}$.

The world seems to have two sectors. The regular sector is the "classical" one. There are proper systems, where the observer's classical mass density (what ever that notion finally may mean) integrates up over ordinary 3-space to the total mass. For an object boosted relative to the observer, the density integrates up to more than the mass like energy increases in Special Relativity. Such a boosted object appears as living partially in the field degree of freedom, while seen from its proper system it lives in spacetime alone. The singular sector (null 4-manifold) is the "quantum" world. The density $m_\phi^2 \phi^* \phi$ of a field living there integrates up over ordinary 3-space to different values, dependent of the observer system. However, there is a subtle - and not yet understood - interaction between the classical and the quantum sector, since this value is quantized in multiples of m_ϕ.