On the spectrum of mass and lifetime

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Abstract

A simple interpretation is given for the new quantum physics resulting from string theory, whose base is an uncertainty relation of the kind $\delta x \geq \frac{\hbar}{\delta p} + G\delta p$, where G essentially is Newton's constant. The algebra is regarded as identic with those of a Hydrogen atom (without fine-structure). As a result, there are restrictions on the mass and lifetime of an object.

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Insights from string and membrane theory suggest a modification of the uncertainty relation of the kind $\delta x \geq \frac{\hbar}{\delta p} + G\delta p$, where G is Newton's constant apart from some numeric factors and with the velocity of light set unity throughout. [1] proposes to replace the unit operator by a momentum dependent expression (see their equation 8). In the following I argue in favor of a much simpler alternative, taking into account that already the traditional term of the uncertainty relation has been a matter of concern. In that respect, a modification towards the SU(2) (locally equivalent to SO(3)) group was discussed [2, 3], based on the mechanism of "contraction" in the sense of Lie group theory.

The algebra so(3) has 3 generators fulfilling $[J_i, J_k] = \epsilon_{ijk} J_k$. The eigenvalue of J^2 is $j(j+1)$. In all the following, all these operators do not act in ordinary space, rather in 3-dimensional phase space and generate infinitesimal transformations of space, momentum, and energy, respectively. Now, our every day experience are phase space rotators with huge j and "magnetic quantum numbers" μ around $-j$. This makes J_3 approximately act as $J_3 \approx -jI$ on all the states under consideration, where I is the unit operator. So, so(3) erroneously is reckognized as the usual algebra of a harmonic oscillator. It shall be emphasized that the situation is even more tricky as if a nonvanishing commutator was contracted to the value zero. A wrong generator is put in place, namely I for J_3. Since I is to be taken on bord, the contraction formally starts from the u(2) algebra as stated in [2]. With J_3 removed from the algebra, the contracted Hamiltonian is defined from $J^2 - J_3^2 = J_1^2 + J_2^2$ and rescaled by a factor of $2j$. Details can be found in [3]. The implications of the entirety of manipulations are grave, in particular what regards the spectrum. In the correct (uncontracted) algebra, the spectrum is bound from both sides. It is worth demonstrating where the ominous "vacuum energy" originates from. If the rescaled Hamiltonian is $H = \frac{\hbar\omega}{2j}(J^2 - J_3^2)$, its effect on states with $\mu = -j + \delta$ approximately is $H|jm> \approx \hbar\omega(\frac{1}{2} + \delta)|jm>$ under the precondition $\delta << j$. So the vacuum energy is a consequence of the eigenvalue of J^2 being $j^2 + j$ rather than j^2 alone.

Furtheron, the rescaled Hamiltonian shall be written as $H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$ with standard meaning of the symbols. Experiment confirms that $\hbar\omega$ is the unit interval of energy. It is the minimal hop in magnetic quantum number which corresponds to the natural number 1. The only further parameter available is the alternative scale of energy, namely the mass m. Since the contraction parameter - that is the quantity whose extreme value is the cause of the misperception - is a pure number, it has to involve the quotient of the two scales. From the above it is clear that j is the contraction parameter, what suggests the identification

$$\frac{m}{\hbar\omega} = j + 1 \ , \tag{1}$$

where here and in the following the minor terms are less conclusive than the leading one. With this, one can estimate $J_3^2 = J^2 - J_1^2 - J_2^2 = j(j+1) - \frac{2jH}{\hbar\omega}$, and the negative root is to be taken. So $J_3 = -\frac{m}{\hbar\omega}\sqrt{1 - \frac{\hbar\omega}{m} - \frac{p^2}{m^2} - \omega^2 x^2}$. This expression takes into account relativistic as well as quantum aspects and has the approximative value $J_3 \approx -\frac{m}{\hbar\omega} + \frac{1}{2} + \frac{p^2}{2m\hbar\omega} + \frac{m\omega x^2}{2\hbar}$.

From here it is not far to the new quantum algebra mentioned at the beginning. For short, a 1-dimensional harmonic oscillator in ordinary space is regarded as a Hydrogen atom (without fine structure) in phase space. I abstain from interpretations in terms of physical mechanisms and only focus on the algebra, like the Hydrogen spectrum had been estimated algebraically before the Schroedinger equation was formulated. But obviously gravitation plays a role, and this role is quite different from the current picture which falls flat when facing the vacuum. In addition to the contraction parameter associated with $\hbar$ in equation (1), there is another such parameter associated with G, namely $\frac{1}{G\omega m}$. This can be brought in connection with the Runge-Lenz vector. Like mass is quantized via equation (1), inverse mass is quantized via this second quantity.

The Hydrogen spectrum is well known. In addition to μ and j comes the principal quantum number n, which is a positive integer associated with the Casimir operator C. The properly normalized value of the Runge-Lenz vector which can be interpreted as the eccentricity of the orbit shall be denoted D. The three operators are related via $C^2 = J^2 + D^2$, where C has eigenvalues $n^2 - 1$. I now identify the eigenvalue of $D^2 + 1$ with $\frac{1}{(G\omega m)^2}$, arriving at

$$\left(\frac{1}{G\omega m}\right)^2 = n^2 - j(j+1) \ . \tag{2}$$

If the relations are rearranged in terms of the parameters, the result is $\left(\frac{m}{m_{pl}}\right)^4 = \frac{(j+1)^2}{n^2 - j(j+1)}$ and $\left(\frac{T}{l_{pl}}\right)^4 = (j+1)^2[n^2 - j(j+1)]$, where $T = \frac{1}{2\pi\omega}$ shall symbolize the duration of one oscillation and the denominators are the Planckian units as derived from G and $\hbar$. As experienced, times are never smaller than the Planck length, while masses can be larger or smaller than the Planck mass. It is satisfactory that in the most fundamental relations the Planck mass and length appear without detour via $\hbar$ and G [4].

For a harmonic oscillator, the interpretation of the quantities involved is retained. What is usually addressed as the "classical" regime by virtue of the correspondence principle is located in a region where δ is large compared to 1, but still tiny compared to j. The state on whom the application of J_1 and J_2 indeed commutes is those with magnetic quantum number zero. There $J_3 = 0$, while $\frac{2H}{m} = J^2 \approx (\frac{m}{\hbar\omega})^2$.

However, the fundamental application of the quantum algebra has not been via the oscillator group, rather via the Heisenberg group. Those group only consists of $[p, x] = -\imath\hbar I$ supplemented by the two trivial commutation relations $[p, I] = 0$ and $[x, I] = 0$. In the limit discussed, it is regarded as relevant for any object associated with a pair of conjungate variables and is completely separated from the dynamics. It can be used in combination with any nonrelativistic Hamiltonian. For this reason, one has abstracted the uncertainty as being a property of phase space as such. Equally, the restrictions on mass and time derived here can be abstracted as properties of (extended) phase space as such. Any object must obey them. For the mass, the meaning is clear. What regards T, the respective quantity that can be assigned to an object at rest relative to the observer is its lifetime.

For second quantization, the analogue case is that of a real scalar field living in time (there is no space in this analogy). The three generators now are associated with the field, its canonical momentum, and the field energy. The mass m_f of the field quantum replaces $\hbar\omega$ from above. The maximum number of field quanta directly is $j+1$ and shall be written as $\frac{\lambda_f m_{pl}}{m_f} = j+1$, where λ_f is a pure number. This corresponds to the former equation (1). In equation (2), the former mass equally is replaced by λm_{pl}, arriving at $\frac{m_{pl}}{\lambda_f m_f} = n^2 - j(j+1)$. As a result, masses of field quanta are bounded from above by the Planck mass via $\left(\frac{m_{pl}}{m_f}\right)^4 = (j+1)^2[n^2 - j(j+1)]$.

References

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