R^0 Gravity

Gerald Vones

Graz, Austria

E-mail: `gev@aonmail.at`

Abstract. It is argued that the symmetry underlying gravitation essentially is like special relativity. The Planck length mediates this symmetry between spacetime-like and field-like degrees of freedom in a flat background space, while in the action solely appears the 4-volume term. A resulting expansion equation for the universe is derived.

PACS numbers: 04.50.+h, 98.80.Bp

Submitted to: *Class. Quantum Grav.*

Cosmological data point to an "admittedly weird composition of the universe" [1], while the problems of quantum gravity [2] and interpretation of quantum physics [3] are open. Recent evidence indicates that general relativity needs adaptation, even in classical segments [4], which destroys all of its majesty and invites to reconsidering its philosophical roots. These aspects are strongly connected and very likely will be their solutions. In the course of history, the formulation of special relativity represented an outstanding event involving insight gained from simple mathematics accompagnied by a substantial change of philosophy. The concepts of space and time were united in a four-dimensional manifold that is invariant under boosts, with the velocity of light, which has dimension, used as the conversion factor. While there is evidence that all physical concepts and even pure numbers are on the same footing [5, 6, 7], Planck's constant and Newton's constant still appear in rather strange symmetries, causing many obscurities. Could it be that adopting the approach used in the context of special relativity represents an appropriate solution?

For gravitation, such a program implies that spacetime and the gravitational field span a space constant under rotations. Since this space lies beyond any gravitational considerations, it is flat, and acts as a non-dynamic and eternal backgroud space. This bold idea, which mitigates many of the deep problems of cosmology and quantum gravity, is supported by fundamental observations. In particular, a mathematical theorem states that a flat embedding space exists (!) for any curved manifold of whatever physical meaning [8]. Furthermore, only physical theories that have overcome the restraint of time are mature enough for full unification with mathematics.

The resulting model is close to the string and brane theories [9], while the interpretation of the degrees of freedom is mutated, with the flat embedding space as the primary entity and the field as degree(s) of freedom in this space. Its pure form presented here appears to be of theoretical interest, irrespective of to what extent it describes the phenomenon of gravitation observed. There is only one relevant free parameter, namely the number of dimensions of the embedding space. However, there are good reasons to start from eight dimensions, four time-like and four space-like. In special coordinates, the line element is:

$$\mathrm{d}s^2 = \sum_{a=1}^{8} \mathrm{d}\Xi_a \mathrm{d}\Xi^a \quad = \quad \sum_{\mu=0}^{3} \mathrm{d}x_\mu \mathrm{d}x^\mu - \ell^2 \sum_{\nu=0}^{3} \mathrm{d}A_\nu \mathrm{d}A^\nu \ , \tag{1}$$

where the greek indices follow the familiar Minkowski metric of flat four-dimensional spacetime, with the velocity of light set to unity. Later, x^0 plays the role of time. This means that x^0, A^1, A^2 and A^3 are time-like, while x^1, x^2, x^3 and A^0 are space-like. s and x have dimensions of length, while the A parameters are pure numbers. ℓ is a conversion factor with dimension of length, for which the Planck length—up to factors of the order of unity—is the only serious candidate.

To arrive at a four-dimensional embedded manifold - the "field manifold" - there are four restricting (embedding) equations. For differentiable embedding relations, the straightforward expression for the four-volume of the field manifold is mostly called the Nambu–Goto action, apart from a proportionality factor. However, since the embedding space is flat, it can be recognized as a consequence of the Gauss theorema egregia, producing intrinsic curvature from flatness, that:

$$V_4 = \int \sqrt{\left| \det \frac{\partial \Xi^a}{\partial \xi^\mu} \frac{\partial \Xi_a}{\partial \xi^\nu} \right|} \, \mathrm{d}^4\xi \ , \tag{2}$$

where ξ symbolizes the degrees of freedom of the field manifold. In the most cases (non-cosmological limit), it is appropriate to identify ξ as x and to write the embedding equations as $A^\nu = A^\nu(x^\mu)$. This and only this is the gravitational field action, apart from a global proportionality factor. Since the curvature scalar R does not appear, it can be called the "R^0 gravity" theory, a rather special case of $f(R)$ theory.

In the appropriate limit, a component of the induced metric does not play the role of Newton's potential, rather A^0 does. The theory in this limit differs from electrodynamics, especially in the lack of the divergence (gauge) term. The resulting correspondence to special relativity is listed in Table 1.

A "vector" resembling character of the A appears as necessary. A "scalar" would exclude massless sources, while a "tensor" makes little sense from the very foundations of the construction. However, there is no vector symmetry, especially there is no conserved vector current. The continuous symmetries present are the eight-dimensional Poincaré symmetry of the embedding space and the four-dimensional diffeomorphism invariance of the phase manifold. The associated conservation law is that for stress energy, with the stress-energy tensor essentially being the metric induced on the field manifold. This fits to gravitation.

The fundamental solutions to the action principle are $\phi(r)$, where ϕ is the "field" depending on the radial component r in a fully Euclidean (here called space-like) space,

Table 1. Correspondence between quantities in special relativity and R^0 gravity. The greek indices follow the Minkowski metric in four dimensions.

Special relativity	R^0 gravity
Time t	Spacetime x^μ
Spatial position $\vec{x}$	Gravitational field A^ν
Velocity of light c	Inverse Planck length ℓ^{-1}
Flat spacetime	Flat embedding space
Trajectory $\vec{x} = \vec{x}(t)$	Field manifold $A^\nu = A^\nu(x^\mu)$
Proper length V_1 of trajectory	Volume V_4 of field manifold
Velocity vector	Induced metric tensor on the field manifold
Mass	"Cosmological" constant (/ Newtons constant)
$\mathrm{d}V_1 \approx (1 - \frac{\dot{\vec{x}}^2}{2c^2})\mathrm{d}t$	$\mathrm{d}V_4 \approx (1 - \frac{\ell^2}{2}\partial_\mu A_\nu \partial^\mu A^\nu)\mathrm{d}^4x$

which, in addition to r, is parameterized by n angles. Hence, the volume of the field manifold is $V_{n+1} \propto \int \sqrt{\left|1 \mp (\frac{\ell\,\mathrm{d}\phi}{\mathrm{d}r})^2\right|}\, r^n \mathrm{d}r$. The sign in front of the field derivative squared depends on whether the field is time-like or space-like. The first integration of the field equation yields:

$$\left(\frac{\ell\,\mathrm{d}\phi}{\mathrm{d}r}\right)^2 = \frac{1}{1 - \left(\frac{r}{r_{max}}\right)^{2n}} \quad , \quad \left(\frac{\ell\,\mathrm{d}\phi}{\mathrm{d}r}\right)^2 = \frac{1}{\left(\frac{r}{r_{min}}\right)^{2n} - 1} \tag{3}$$

for time-like or space-like ϕ, with manifold of maximum and minimum volume, respectively. r_{max} and r_{min} are constants of integration. These values of r represent branch points where the derivative of the field is infinite. The two solutions can be regarded as an analytical continuation of each other, inside and outside the sphere where the derivative of the field is singular called the "terminator". Figure 1 gives a sketch of the solutions.

These fundamental solutions can be used for two important configurations. The first is the universe in the approximation of spatial homogeneity and isotropy. In this case, all four space-like degrees of freedom of the eight-dimensional embedding space span the basic Euclidean space, where 3-spheric coordinates are to be introduced with r as the radial degree of freedom. This space is mapped into the timelike field $x_0 \equiv t \cong \ell\phi$, while the other three time-like degrees of freedom of the embedding space are zero. Thus, the inner solution for $n = 3$ is used, with r corresponding to the radius of the universe as it appears in the Robertson–Walker line element, and t corresponding to time (we can set $\phi(0) = 0$). Reinterpretation of time as the independent variable leads to the expansion equation of the universe $(\frac{\mathrm{d}r}{\mathrm{d}t})^2 = 1 - (\frac{r}{r_{max}})^6$. To complete the picture, we can explicitly add a source term proportional to ${r_{max}}^3$ but located at $r = 0$, unveiling "vacuum" and "cosmic fluid" as two sides of the same coin, with total Hamiltonian of zero. It should be noted that, in terms of t, the velocity of light is constant in the embedding space, while as a consequence of simple geometry, the embedded universe shows a variable speed of light [10, 11]. The data on the age of the universe and the Hubble constant

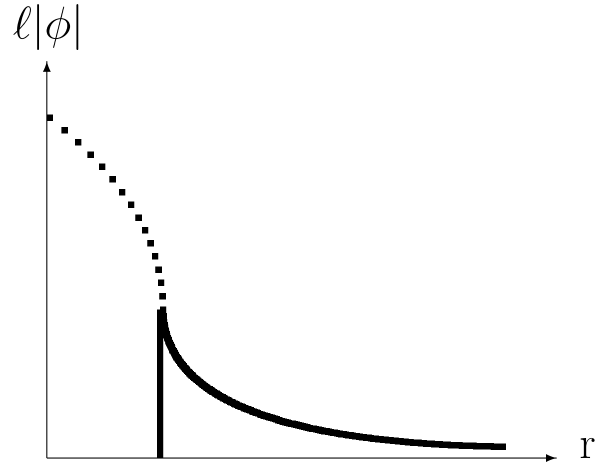

Figure 1. Fundamental solution for the inner and outer region, for specific choices of the respective branches as well as the second constants of integration. The vertical line symbolizes a source term contributing with opposite sign to the volume of the field manifold. The line element in this plane is $\mathrm{d}s^2 = -\mathrm{d}r^2 \pm \ell^2 \mathrm{d}\phi^2$ respectively, where the overall sign is a convention. At $r = 0$ the slope is 45 degrees and becomes infinite at the terminator, while $\phi \asymp \frac{const}{r^{n-1}}$ for $n > 1$.

provide evidence that these are actually estimated in terms of t rather than in terms of the proper time within the universe, in contrast to the current definition of units [12]. This is strongly relevant for the interpretation of other cosmological data as well. The solution of the field equation exhibits 3-spheric symmetry near $r = 0$ and thus is of low entropy [13], even if this is not postulated a priori. The homogeneous field equation is $\frac{\triangle\phi}{[1-(\ell\nabla\phi)^2]^{3/2}} = 0$, where the differential operators act in 4 fully euclidean dimensions. This essentially is the Laplacian equation in 4 dimensions. The partial waves behave as $\propto r^L$, where L is the angular momentum quantum number. So, for $L > 1$ they are not causal in the embedding space, i.e. $\frac{\mathrm{d}r}{\mathrm{d}t} > 1$, while $L = 1$ is not self-consistent.

The other fundamental case involves a massive non-rotating point source described in its proper rest frame. This static configuration is translation invariant along x^0, while the other time-like degrees of freedom vanish. Thus, x^1, x^2 and x^3 build the basic Euclidean space, which is mapped into A^0. The outer solution for $n = 2$ is used, with $\phi \equiv A_0$ asymptotically identified with the Newtonian potential. Hence, $r_{min} = \sqrt{\ell GM}$, where G is Newton's constant and M is the mass of the point source. Furthermore, $\ell|\phi(r_{min})| = br_{min}$, where $b = \int_1^\infty \frac{\mathrm{d}x}{\sqrt{x^4-1}} = 1,31\ldots$. The second constant of integration has to be set to $\phi(\infty) = 0$. The generalized solution, depending on the angles as well, reflects the angular momentum of the source.

If the point source solution is unified with the cosmological solution, then the radial degree of freedom of the universe essentially plays the role of the Newtonian field of the particles. This is not discussed here, and the cosmological curvature is neglected in the following.

From the point source solution, assembly of an ensemble of sources is straightforward. In line with the symmetries of the theory, the source terms must be four-dimensional subvolumes of the embedding space as well. This makes the total action $S \propto V_4^{field} + \sum_n V_4^n$, where the sum is over all sources. We can locate the divergence of the field flux at the two-dimensional terminators moving through space, which realizes the old idea of matter existing at singularities of the field in place of the smeared out stress energy tensor. Since the terminator is locally orthogonal to the trajectory (by definition), as well as the field jumps (see the thick vertical line in Figure 1), any summand of $\pm \ell A_\mu dx^\mu$ is an infinitesimal two-dimensional subvolume of the embedding space orthogonal to the terminator. In this expression, the sign is irrelevant for the embedding symmetry, since the A parameters change sign simultaneously; in particular, $\pm A^0 = -|A^0|$ if the volume of the field manifold is counted as positive, so the field manifold and the source term contribute with opposite sign (formally, all $V_4^n < 0$ in the sum above). The free particle term can be derived from a change of the additive constant of integration —with sign so that the force is attractive— adding u_μ to the jump in A_μ. Here u is the relativistic four-velocity of any point at any terminator derived from the coordinate velocity $v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} := (1, \vec{v})$. Thus, each source of non-vanishing mass contributes

$$V_4 = \ell \int (u_\mu \pm A_\mu) \mathrm{d}x^\mu \mathrm{d}V_2^{termi} \,. \tag{4}$$

As long as the terminator area is constant in the respective particle frames, mass acts as a conserved quantity. However, when the sources interact in such a way that the terminator areas vary, then non-conservation of the current is relevant.

In the Newtonian approximation, the terminator area is $4\pi\ell GM$ and $\vec{v}$ is regarded as constant all over the terminator; thus, the free particle term located at the particle world-line becomes $V_4 = 4\pi\ell^2 GM \int \sqrt{1-\vec{v}^2}\mathrm{d}x^0$, which is further reduced to $-4\pi\ell^2 GM \int \frac{\vec{v}^2}{2}\mathrm{d}x^0$. The volume of the field manifold is approximately $\int \sqrt{1+(\ell\nabla A^0)^2}\mathrm{d}^4x$, which is further reduced to $\ell^2 \int \frac{(\nabla A^0)^2}{2}\mathrm{d}^4x$. The force is mediated by the coupling term $\pm 4\pi\ell^2 GM \int A^0 \mathrm{d}x^0$.

The above construction assumes that the field-like directions are the same everywhere. Further investigations are required to determine whether a more general case incorporates other interactions than gravity.

The proportionality factor between the four-volume and the action, the quasi "cosmological constant", is irrelevant, even for the expansion equation of the universe apart from the relation to r_{max} via the source term of the universe. There are many indications (the field energy of a point source, which is proportional to $M^{\frac{3}{2}}$; the different sign of the volume terms of different parts of the manifold; the different extremum associated with inner and outer regions; the saddle point character of the cosmological solution if homogeneity and isotropy are not postulated) that fit a purely imaginary action rather than a real one.

References

[1] Padmanabhan T 2005 Dark energy: Mystery of the millennium. In: *Proceedings of the Albert Einstein Century International Conference, Palais de l'Unesco, Paris, 18–23 July 2005*, arXiv:astro-ph/0603114

[2] Smolin L 2003 arXiv:hep-th/0303185

[3] t'Hooft G 2006 arXiv:quant-ph/0604008

[4] Bekenstein JD 2004 *Phys. Rev.* **D70** 083509; Erratum 2005 **D71** 069901 (arXiv:astro-ph/0403694 and arXiv:astro-ph/0412652)

[5] Sikivie P 1982 Dynamical symmetry breaking, In: *Proceedings of the XXI Internationale Universitaetswochen für Kernphysik, Schladming, Austria, Feb 25–March 5 1982* (Acta Phys. Austriaca Suppl. 24)

[6] Duff MJ 2004 arXiv:hep-th/0208093

[7] Dewdney AK 1999 *A Mathematical Mystery Tour* (New York: John Wiley and Sons)

[8] Pavšič M and Tapia V 2001 arXiv:gr-qc/0010045

[9] Pavšič M 2001 *The Landscape of Theoretical Physics: A Global View* (Dordrecht: Kluwer Academic Publishers)

[10] Magueijo J 2003 *Rep. Prog. Phys.* **66** 2025 (arXiv:astro-ph/0305457)

[11] Rañada AF and Tiemblo A 2006 arXiv:gr-qc/0602003

[12] Organisation Intergouvernementale de la Convention du Mètre 1998 *Le Système International d'unités* 7th edition

[13] Penrose R

1979 Singularities and Time-Asymmetry, In Hawking SW and Israel W (Editors) *General Relativity: An Einstein Centenary* (Cambridge University Press)

2005 *The Road to Reality: A Complete Guide to the Laws of the Universe* (Oxford University Press)