R^0 Gravity

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Abstract

It is argued that the historic step towards Special Relativity should be plagiated in the context of gravitation. The Planck length mediates the symmetry between spacetimelike and fieldlike degrees of freedom, while in the action appears the 4-volume term, but no curvature term. The resulting expansion equation of the universe is derived.

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About one century after the golden age of physics, there appears to be an "admittedly weird composition of the universe" [1], a fully consistent theory of quantum gravity is not yet reached despite of promising progress [2], and the quantum debate as such is continued for good reasons [3]. Furthermore, there is growing evidence that General Relativity needs adaptions even in classical segments [4]. All these aspects are so strongly connected that they can be regarded as one big problem. History of physics teaches that the solutions to big problems are surprisingly simple, but go hand in hand with the change of some persistent philosophical positions. There is an outstanding event when insight was gained, this is the formulation of Special Relativity. While former physics had distincted strictly between space and time, they then were reckognized to be on the same footing spanning a 4-dimensional manifold invariant under boosts, with the dimensionful velocity of light as the conversion factor. There is overwhelming evidence that all physical quantities and even pure numbers are on the same footing [5] and that only pure numbers correspond to observable effects [6] [7]. However, Plancks constant and Newtons constant so long appear in rather strange symmetries, what is the cause of all the obscurities. Couldn't it be that plagiating the step towards Special Relativity teaches how to overcome even Einsteins own theory of gravitation?

Such a program implies that spacetime and the gravitational field have to appear on the same footing. Consequently, they span a space which lies beyond any gravitational aspects. So this space is nondynamic, eternal, flat, much against the current philosophical mainstream. But with the following, based on well established mathematics and not far from the constructs of strings and branes [8], I try to make transparent that there is no conceptual conflict with mathematics and physics - quite the opposite. Especially, the proven theorem stating that a flat background space exists(!) in the sense that is any intrinsic curvature can be produced from appropriate flat embedding [9] is not adequately reflected by General Relativity. Furthermore, only a physical theory which has overcome the prison of time and dynamics is mature for a full unification with mathematics, which enjoys this freedom - three plus four has not been seven or will be, it simply is.

The resulting classical theory of gravitation has all the properties one can expect: A fundamental symmetry mediated by a "Planckian" quantity, the lack of relevant free parameters apart from the number of dimensions and the constants of integration, nonlinear coupling emerging from an action which could not be more transparent and simple, and many more. If it failed to describe the physical phenomenon of gravitation, this could at least be a helpful "no go" theorem. As a starting point, regard a 8-dimensional flat embedding space with 4 timelike and 4 spacelike degrees of freedom. In a special set of coordinates, the lineelement is

$$ds^2 = \sum_{a=1}^{8} d\Xi_a d\Xi^a \quad = \quad \sum_{\mu=0}^{3} dx_\mu dx^\mu - l^2 \sum_{\nu=0}^{3} dA_\nu dA^\nu \ , \tag{1}$$

where the greek indices are to be raised and lowered by virtue of the familiar Minkowski metric of flat 4-dimensional spacetime with the velocity of light set unity. This means, x^0, A^1, A^2, A^3 are timelike, while x^1, x^2, x^3, A^0 are spacelike (x^0 shall lateron play the role of time and thus is timelike by definition). s and the x shall have the dimension of length, while the A are pure numbers. l is a conversion factor with the dimension of a length, for which the Planck lenght - up to factors of order of magnitude unity - is the only serious candidate. The chosen number of fieldlike degrees of freedom (A) may not be the ultimate wisdom, but as will be visible from the following there is much plausibility behind as long as pure gravity is regarded.

To arrive at a 4-dimensional embedded manifold (the "field manifold") there must be 4 restricting equations. As long as these embedding relations are differentiable, there is a straightforward expression for the 4-volume of the field manifold. Apart from a proportionality factor, in the literature this is mostly called the Nambu-Goto action. But since the embedding space is flat, one can reckognize it as Gauss' theorema egregia with the great and sanative property to produce intrinsic curvature from flatness

$$V_4 = \int \sqrt{\left| \det \frac{\partial \Xi^a}{\partial \xi^\mu} \frac{\partial \Xi_a}{\partial \xi^\nu} \right|} \, d^4\xi \ , \tag{2}$$

where the ξ shall symbolize the 4 degrees of freedom of the field manifold. In most cases, it will be technically appropriate to identify the ξ with the x and to write the embedding equations as $A^\mu = A^\mu(x^\nu)$. Now, this and only this shall be the gravitational field action apart from a global proportionality factor. Since the curvature scalar R does not appear, one can call the theory "R^0 Gravity", a rather special case of a R^n theory.

The central point is that in an appropriate limit not a component of the induced metric will play the role of Newtons potential, rather one of the A itself. So, the conversion factor l mediates between "spacetime" x and the "field" A.

This has some similarities with a theory of a vector field living on flat spacetime. However, the only continuous symmetries present are the 8-dimensional Poincaré symmetry of the embedding space and the 4-dimensional diffeomorphism invariance of the phase manifold. The associated conservation law is those of stress-energy defined in the usual way, with the stress-energy tensor being identic with the induced metric on the field manifold, while there is no conserved vector current. So, these symmetry fits to the description of gravitation. Furthermore, the embedding is invariant under change of sign of all the A.

As fundamental solutions to the action principle shall be understood one field component living on a $n+1$ dimensional fully euclidean (shall be called spacelike) space, the configuration being spherically symmetric i.e. constant along the n angles in that flat space. This means, there is one restricting equation in a $n+2$ dimensional embedding space and the volume of the embedded manifold is $V \propto \int \sqrt{\left|1 \pm (\frac{l\,d\phi}{dr})^2\right|} r^n dr$, where ϕ is the "field" and r is the radial coordinate in the euclidean space. The sign in front of the field derivative squared depends on whether the field is spacelike or timelike. The first integration of the field equation yields

$$\left(\frac{l\,d\phi}{dr}\right)^2 = \frac{1}{1-\left(\frac{r}{r_{max}}\right)^{2n}} \qquad , \qquad \left(\frac{l\,d\phi}{dr}\right)^2 = \frac{1}{\left(\frac{r}{r_{min}}\right)^{2n}-1} \tag{3}$$

for timelike or spacelike ϕ, respectively. In the first case this is a maximum manifold, while in the second case it is a minimum manifold. r_{max} and r_{min} are constants of integration. At those values of r the derivative of the field becomes infinite, while the field itself stays finite. The two solutions can be regarded as analytic continuation of each other, inside and outside the singular sphere which shall be called the "terminator". In the figure, the solutions are sketched for specific choices of the second constants of integration (which can of course be changed).

This fundamental solution can be used for the two most popular configurations. The first is the universe in the approximation of spatial homogeneity and isotropy. In this case, all four spacelike degrees of freedom of the original 8-dimensional embedding space are to be used as the basic euclidean space. Living on it is x^0, while the other three timelike degrees of freedom of the embedding space are zero. So, the inner solution for $n=3$ is to be used, with r corresponding to the radius of the universe as it appears in the Robertson-Walker lineelement, and $l\phi := t$ corresponding to time (one can set $\phi(0)=0$). To complete the picture, one can explicitely add a source located at $r=0$ unvealing the "vacuum" and the "cosmic fluid" as two sides of the same medal with total Hamiltonian zero. Reinterpreting time as the independent variable, one arrives at the expansion equation of the universe

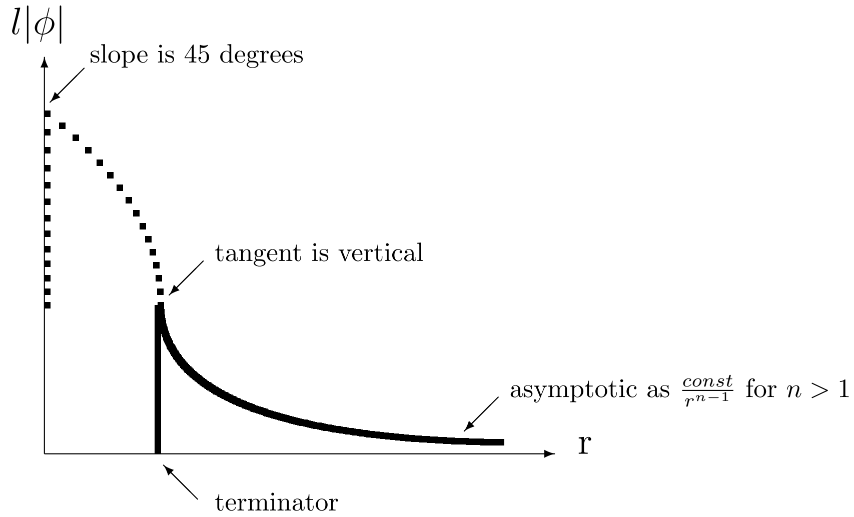

Figure 1: The fundamental solution for the inner and the outer region. The vertical lines shall symbolize the respective source term. The lineelement in this plane is $ds^2 = -dr^2 \pm l^2 d\phi^2$ respectively (the overall sign is a convention).

$\dot{r}^2 = 1 - (\frac{r}{r_{max}})^6$, where the overdot means derivative w.r.t. time. Nota bene, in terms of this time the velocity of light is constant in the embedding space, while as a consequence of simple geometry the embedded universe shows a variable speed of light [10]. The data on the age of the universe and the Hubble constant give strong evidence that they factually are estimated in terms of t rather than in terms of the proper time within the universe, against the current definition of units [11]. This example gives an eloquent warning what regards the interpretation of cosmological data. This model of the universe modifies the deSitter solution to the field equations of General Relativity invariant under SO (4,1) in an appropriate way. Remarkably, the solution of the field equation exhibits 3-spheric symmetry near $t = 0$, even if this is not postulated a priori. The different 3-spheric harmonics $\phi(angles)$ are coupled to $\phi(r)$ in a well defined way. Only for ϕ independent of the angles the solution is causal in the embedding space, i.e. $\dot{r} \leq 1$.

The other central case is a nonrotating point mass in noncosmological approximation described in its own rest frame. Here, the configuration is static what means translation invariant along x^0, while the other timelike degrees of freedom again vanish. Thus, x^1, x^2, x^3 build the basic euclidean space and A^0 is living on it. The outer solution for $n = 2$ is to be used, with ϕ asymptocally identified with the Newtonian potential. From this one immediately gets $r_{min} = \sqrt{lGM}$, where G is Newtons constant and M is the mass of the point source. Furthermore, $l|\phi(r_{min})| = br_{min}$, where $b = \int_1^\infty \frac{dx}{\sqrt{x^4-1}} = 1,31\ldots$. The second constant of integration has to be set $\phi(\infty) = 0$. In this case, the generalized solution which depends on the angles as well, reflects angular momentum of the source. If the unification with the cosmological solution is sought, then the field of the point source coincides with the radial degree of freedom of the universe.

From the point mass solution one can assemble an ensemble of sources with the cosmological curvature neglected. To be in line with the symmetries of the theory, the source terms must be 4-dimensional subvolumes of the embedding space as well. This makes the total action $S \propto V_4^{field} + \sum_n V_4^n$, where the sum runs over all sources. To this end, one can locate the divergence of the field flux at the 2-dimensional terminators which move through space, what realizes the old idea of matter living at the singularities of the field. Since the terminator is locally orthogonal to the trajectory (by definition) as well as the field jumps (from the value as approached from outside to zero inside the terminator; see the thick vertical line in figure 1), any summand of $\pm lA_\mu dx^\mu$ is an infinitesimal two-dimensional subvolume of the embedding space orthogonal to the terminator. In this expression, the sign is irrelevant from the symmetry of embedding, since the A change sign simultaneously; especially $\pm A^0 = |A^0|$. The only element not automatically part of the field manifold is the free particle term, but this can be supplemented explicitely by adding u_μ to the jump of A_μ. Here u is the relativistic 4-velocity with $u^2 = 1$ of any point at any terminator resulting from the coordinate velocity $v^\mu = \frac{dx^\mu}{dx^0} := (1, \vec{v})$. So each source of nonvanishing mass contributes

$$V_4 = l \int (u_\mu \pm A_\mu) dx^\mu dV_2^{termi} \ , \tag{4}$$

where n is running over all the point sources present. As long as the terminator area is constant in the respective particles frame, the coupling is like in a vector theory and mass acts as a conserved quantity. However, when the sources interact in a way that the terminator areas vary and entropy enters the scene, then the nonconservation of the current is relevant.

The above construction assumes that the fieldlike directions are the same everywhere. This is a reflection of our factual world, where one can tell rather clearly what is field and what is spacetime. However, this is not the general case and it will in particular have to be investigated whether this incorporates other interactions than the gravitational. The unification with the cosmological solution is rather straightforward.

In the Newtonian approximation, the terminator area is $4\pi lGM$ and $\vec{v}$ is regarded as constant all over the terminator, thus the free particle term located at the particles world line becomes $V_4 = 4\pi l^2 GM \int \sqrt{1-\vec{v}^2} dx^0$, what is further reduced to $-4\pi l^2 GM \int \frac{\vec{v}^2}{2} dx^0$. The volume of the field manifold approximately is $\int \sqrt{1+(l\nabla A^0)^2} d^4x$, what is further reduced to $\frac{l^2}{2} \int (\nabla A^0)^2 d^4x$. So, the relative sign between the two terms is such that the force is attractive. This force is mediated by the coupling term $\pm 4\pi l^2 GM \int A^0 dx^0$.

I avoided to give a value for the proportionality factor between the 4-volume and the action. To some extend this quantity could be addressed as the cosmological constant, however its value is irrelevant even for the expansion equation of the universe apart from the relation to r_{max} via the source term of the universe. Though there is a conserved stress-energy tensor, the interpretation of the so described quantitity is not trivial. For example, when the field energy density of a single point source is integrated up (minus the divergent contribution of a flat field manifold), the result obviously is proportional to $M^{\frac{3}{2}}$ rather than proportional to M. Furthermore in the general case, regions of the universe are inner regions of the cosmological solution, while they are outer regions of the point sources present. So, they are part of a maximum and a minimum surface at the same time. Indeed, if the postulate of homogeneity and isotropy is given up, then the expansion equation is a saddle point of the field equations - and this fits to a purely imaginary action rather than to a real one.

One can act on the flat embedding space with standard quantization methods if one reckognizes it as a phase space with the x and the respective A as the conjugate variables. However, this should not inhibit attempts to achieve a deeper understanding of quantization by reformulating the symmetry mediated by Plancks constant in an appropriate manner. As an intermediate step one can reckognize that the background space is not only eternal and static what regards its linelement, but equally well what regards its volume element. What is to be quantized instead, is the field manifold in whose place should come a phase trajectory. After clarifying the role of the curve parameter, the result should be a countable subset (a cloud of points) of the continuous embedding space.

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References

[1] T. Padmanabhan, Plenary talk at Albert Einstein Century International Conference at Palais de l'Unesco, Paris, France, 18-23 July, 2005; to appear in the Proceedings; arXiv:astro-ph/0603114

[2] L. Smolin, arXiv:hep-th/0303185

[3] G. t Hooft, arXiv:quant-ph/0604008

[4] J.D. Bekenstein, Phys.Rev. D70 (2004) 083509; Erratum-ibid. D71 (2005) 069901, arXiv:astro-ph/0403694 and PoS JHW2004 (2005) 012, arXiv:astro-ph/0412652

[5] P. Sikivie, Dynamical Symmetry Breaking (introduction), Proceedings of the XXI. Internationale Universitaetswochen fuer Kernphysik, Schladming, Austria, Feb. 25 - March 5, 1982, Acta Physica Austriaca Suppl. 24, 1982

[6] M. J. Duff, arXiv:hep-th/0208093

[7] A. K. Dewdney, A Mathematical Mystery Tour, John Wiley and Sons, New York (1999)

[8] M. Pavšič, The Landscape of Theoretical Physics: A Global View, Kluwer Academic Publishers (2001)

[9] M. Pavšič, V. Tapia, arXiv:gr-qc/0010045

[10] J. Magueijo, Rept. Progr. Phys. 66 (2003), 2025, arXiv:astro-ph/0305457

[11] Le Système International d'unités, Organisation intergouvernementale de la Convention du Mètre, 7e edition 1998