Ultimate Quantization

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Abstract

It is argued that quantization means that $\imath$ actually is a real integer, namely a power of 2 characterizing the number of degrees of freedom of nature.

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There is some hope that the not yet achieved theory of quantum gravity thence will help to comprehend quantum physics as such. Currently, while non-gravitational quantum physics has become an undisputed and highly productive tool for practical purposes, the quantum debate has gained new facettes [1]. But how big is the chance that the question marks once will be lifted? In the following I argue that this yet can be achieved. I call the solution "ultimate quantization" since it leaves behind all the dimensionful physical concepts known. Only mathematical objects - pure numbers, complex though - are involved.

That phase space has a symplectic structure already can be learned from classical Hamiltonian mechanics. Put in simple words, symplectic structure means a purely skew-symmetric metric. When used for the description of physical systems the entries have to be purely imaginary to be hermitean. For the most simple choice of coordinates and only one degree of freedom, the position vector in phase space has as components a canonical degree of freedom and the associated conjugate momentum $\xi = \binom{q}{p}$. The symplectic structure reads $g = \frac{\imath}{\hbar}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Thus the line element squared, which in this case yet is the volume element measured in pure numbers is

$$\mathrm{d}s^2 = g_{ij}\mathrm{d}\xi^i\mathrm{d}\xi^j = \frac{\imath}{\hbar}\left(\mathrm{d}q\,\mathrm{d}p - \mathrm{d}p\,\mathrm{d}q\right)\ . \tag{1}$$

As the heart of quantization one regards the fact that the terms in the paranthesis doe not cancel out. The symplectic structure is as given above rather than zero. But this cannot be the crux of the matter. Symplectic structures can be defined in mathematics to measure the oriented volume of manifolds, without any aspect of quantization.

The dimensionfulness of q and p is not the point either. One has to devide them by the Planck length and the Planck mass respectively to arrive at the adequate pure numbers, while the mystery remains exactly the same. The clue only can lie in $\imath$. The imaginary unit makes no reckognizable sense when the volume of a manifold is measured, while for the purpose of quantum physics its apperance is inevitable.

To see how $\imath$ works, it is sufficient to regard the most elementary configuration: First quantization of a nonrelativistic spinless particle moving in a conservative external field. The Schroedinger equation for the wave function $\phi = r\exp\imath s$ decomposes into a real and a purely imaginary part, so there emerge the two well known equations

$$\hbar\frac{\triangle r}{r} + \frac{(\nabla s)^2}{2m} = E - U \tag{2}$$

$$\imath\,\nabla(r^2\nabla s) = 0\ , \tag{3}$$

where m is the mass of the particle, E is its constant total energy, while U is its potential energy which may vary as a function of position. The first equation is the classical Hamilton-Jacobi equation apart from the first term - the only place where Planck's constant appears. The second is the conservation law for the current. I retained the imaginary unit there to remind that this is the imaginary part of the Schroedinger equation. One may question whether the wave function is complex at all, since this is related to an electric charge carried by the particle. But it is well known from the classical experience that a conserved current of particles exists, irrespective of whether they are charged or not. Even the electromagnetic field, which is real, can be treated in such way, the current then being intepreted in terms of photons. This current is not rigorously conserved since photons can be created and destroyed, but in an "adiabatic" approximation conservation holds.

In the classical case where $\hbar$ is set zero, one simply solves the first equation an puts the particle on one of the trajectories generated by the vector field ∇s. This solves the system of equations in a consistent way. When $\hbar$ has a nonvanishing value, however, a particle located on a trajectory is not a solution to the system of equations. The de Broglie pilot wave theory with the clarifications added by Bohm shows that one nevertheless can continue the classical picture to the quantum case, except of the trajectories becoming very strange. They are curly and the gradient of s, which is just the momentum, may even grow so large that the particle becomes superluminal. This extends the problem substantially since a relativistic view would be necessary what in turn leads to a many-particle interpretation and so on.

I argue that the conclusion to be drawn is so radical that all of the said aspects are irrelevant: The system of equations above, and corresponding equations for any quantum theory relativistic or not, first quantization or second, involving Newton's constant or not, is fallacious. It contains more information than there is in the physical world. In the above, there are not two equations, there actually only is one. $\imath$ is not imaginary, it actually is real:

$$\imath \in \mathbb{R} \, . \tag{4}$$

The weird this may appear at a first glance, the trivial it can be. It is realized in any every-day c-bits computer if some part of the memory is assigned to the integer real part of complex numbers, while the immediately consecutive part of the memory is assigned to the integer imaginary part of the number. If there are n bytes reserved for real and imaginary part each, the real integer 2^n will be indistinguishable from the purely imaginary integer $\imath$ and so on. The handling of the signs as well as of negative powers of 2, over- and underflows are pure technicalities and actually correctly implemented in any computer.

Is this a logic circle, to use a digital computer for the explanation? May be, but nevertheless it is correct. The information of nature is a finite integer as I conclude in [2] on the basis of an information-theoretic approach stripped from any concrete physical concept. A bit - a two-valued variable - is the quantum of information. The eventual conclusion is

$$\imath = 2^n \tag{5}$$

where n is some huge integer characterizing nature as we experience it.

One may wonder how this can go together with the way we have used $\imath$ without problem. But this is just a special case of the many where we as conscious beings can handle objects that only exist in our imagination.

References

[1] 't Hooft G, arXiv:1405.1548 [quant-ph] (2014) and references therein

[2] Vones G, Why are there Laws of Nature?