Units are unique

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Abstract

It is argued that the set of physical units is uniquely determined from the set of dimensionful concepts introduced. The basic concepts of spacetime and energymomentum can be interpreted as degrees of freedom in complex phase space equipped with the Minkowski metric.

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Economic life requires to compare apples and oranges in terms of a unique currency - money talks. Intermediate units drop out there and then where they are introduced. With the minor exception of a global rescaling of the monetary unit by the government, the system has no degree of freedom.

The question about physical units and dimensionful fundamental constants [1, 2] appears to be of the same kind. While space and time had been regarded as fundamentally different, Special Relativity teaches that the velocity of light is a rigid exchange rate if introduced appropriately into the formulae, so actually there is a common currency.

Quantum physics teaches that Planck's constant is a rigid exchange rate between action as the fundamental dimensionful entity, and information which is a pure number by its very meaning. Hence, like reductionism [3] sees chemistry essentially as physics of atomic bonds, *physics can be seen as mathematics of information*. The concept of information can well exist independent of any observer, as I argue elsewhere [4].

While presumably the majority of physicist would agree on the key role of information, noone has persued this towards the consequences. The reason is that converting all physical quantities to pure numbers receives a dramatic setback when Newton's constant G enters the scene. In the Hilbert-Einstein action, Newton's constant converts the curvature scalar with dimension of inverse length squared to a Lagrangian density which has the usual dimension of energy per 3-volume. This role is much obscure compared to the transparent role of the velocity of light inside a line element as well as from the transparent role of Planck's constant as a unit of the phase space volume.

It is quite straightforward to conjecture how Newton's constant rather should appear, namely in a line element involving spacetime and energymomentum in a space of extended number of dimensions. Two cases are to be distinguished. First, the degrees of freedom involved are not conjugate variables, what brings no further problems. Second, the degrees of freedom are conjugate variables governed by the symplectic structure of phase space. This is more subtle, however mathematics opens a clear possiblity. Phase space is intrinsically complex as is manifested by the ladder operators $a, a^+ = p \pm \imath x$. In this expression the symbols can have standard meaning, while I already used Planckian units such that all the quantities are pure numbers. A complex space $\mathcal{C}^n$, where n is the number of dimensions, can be equipped with a hermitean metric. This makes it a Kaehler manifold where Riemannian and symplectic structure coexist through the complex structure $h = h_{ik} \mathrm{d}z^i \otimes \mathrm{d}z^k$. For brevity, I shall always speak about the "metric" what refers to h_{ik}.

On this basis, the following - not exhaustive - list of consequences from

bringing together the ideas of information theory and reductionism, can be dealt with:

- Information is a pure number. Dimensionful physical concepts are unnecessary and had been introduced because of ignorance, but can well serve practical purposes where no direct connex to information is needed. There are exactly as many fundamental dimensionful physical units as dimensionful concepts introduced.

- Pure numbers as elements of the respective real or complex fields are not scale invariant. Information as the number of degrees of freedom of a system is absolute and cannot be redefined by a change of scale. The apparent freedom to choose units is a chimera reflecting the incompleteness of current theoretical physics.

- Pure numbers are prior to physics, as in particular the notions of space and time are unknown to mathematics (eleven minus seven has not been four or will be somewhere, it simply is). Physics exists and is to be formulated with reference to a nondynamic prior background.

- The background defines the zero point of information. By itself, it is void of information to the maximum possible. Hence this background is a flat space, unbounded and free of topological effects. Even the coordinates are prior cartesian since the generation of the (pseudo)unit matrix field needs the shortest code.

- The global symmetries of the background unambigously gauge and transport units. Embedded manifolds may show nontrivial intrinsic and/or extrinsic curvature, producing differential geometric effects as well as topological effects.

To conclude, there is a prior space $\mathcal{C}^n$ with a sufficient number of dimensions, that can be interpreted as extended phase space. The metric is (pseudo-)Euclidean if the degrees of freedom are identified with the physical quantities as usual (else, there is no difference between Euclidean and pseudo-Euclidean in complex space). First one may ask whether the continuum is the appropriate basis since nature is known to be quantized. But this is just the point: Mathematics well knows the continuum. The pre-existence of the embedding continuum together with the attached prior metric exactly offers the possibility to embed something that is really quantized. Integrability implies an embedded torus, superintegrability implies an embedded 1-dimensional phase trajectory, while quantization implies a 0-dimensional

(countable) embedded cloud of points with mutual distances defined. A discussion on the mightiness can be found in [4]. Second, one may ask about the prior coordinates. If, at the continuum level, the physical structure is a torus, then torus coordinates should be preferred rather than cartesian ones. This is indeed the case, however the torus yet is the physical entity. A torus preferres a point in any 2-dimensional subspace of phase space, and this is the minimum amount of information to be added to the background.

The degrees of freedom of the prior structure have to comprise those concepts which have had assigned physical dimensionality, like the non-gravitational fields (usually assigned the dimension of a nontrivial power of mass). A lineelement involving spacetime and fields is not a new idea. In particular, the Born-Infeld action for electrodynamics implicitely is based on such symmetry, despite of the fact that the electromagnetic field is not so straightforward to handle, first being a spin 1 part of a vector, second being massless. Extensions beyond a $\mathcal{C}^n$ may be necessary for Fermionic fields, and then there is the very delicate case of the gravitational field. But all this is beyond the scope of this note which only aims at arguing that units are unique. Rather, here one can put the primary focus on the familiar measuring devices which are "objects", whose positions and momenta act as degrees of freedom. A clock, in Special Relativity sweeping out spatial distances via motion, here can sweep out energy by aquiring varying potential energy from a field it couples to.

The plausible this consideration is, the diametrally it stands against General Relativity in almost every fundamental aspect. So, can it be right? I argue that the answer is yes. First, history of physics teaches that eventually those premises turn out as inappropriate which have been regarded as the most firm ones, both for philosophical reasons as well as for their apparent quantitative success. Background-independence is the current most pertinent pre-judgement. Efforts are made to hide the background where it is present naturally, like in string and membrane theory. From the above one can conclude that strings and membranes are well adequate to describe nature at the continuum level if the degrees of freedom are given a new interpretation. Second, the current description of gravitation is responsible for theoretical physics being incomplete. The concept of a fundamental dynamic metric is quantitatively successful in some parameter region, however it falls flat when facing the vacuum and the quantum. Again from the above, one can conclude that the vacuum well has its natural energy density, that is one Planck mass per Planck volume. Third, the current theory of gravitation exactly is a description where constant gradients of the field can be transformed away, only the mechanism how this comes along yet is different.

The above opens the door for a unification of such behaviour. This eventually removes the absolute character of the spacetime geometry which again implies dramatic consequences, in particular for the Unruh effect which then is naught.

Even if the theory of gravitation was left untouched, there would emerge an illustrative consequence from the above. The role of the velocity of light is ultimately transparent if spacetime is flat, while it remains obscure if spacetime is curved as the discussion on a variable speed of light (VSL) [5] reflects. Now a mathematical theorem proves that a flat embedding space of sufficient number of dimensions *exists* (no other way of existence but this) so that any intrinsic - and extrinsic, of course - curvature can be produced from embedding [6]. To treat information correctly, the "velocity of light" must be set globally unity in the flat embedding space. Consequently it varies in any embedded manifold other than an intrinsically flat one. The opening angles of the null cones - these are pure numbers - emerge from the embedding equations whose role exactly is to encode the shape of the manifold. This means that the current declaration due to which the velocity of light is constant inside spacetime [7] cannot be retained if theoretical physics shall be made complete.

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